Section 5.2 – The Number of Elements in a Finite Set

Let \( A \) be a set, then \( n(A) \) is the number of elements in set \( A \).

**Example 1:** Let \( A = \{1, 2, 3, \ldots, 19, 20\} \) and \( B = \{q, s, t, v\} \). Find:

1. \( n(A) = 20 \)
2. \( n(B) = 4 \)
3. \( n(\emptyset) = 0 \)

Given two sets \( A \) and \( B \).

1. If \( A \) and \( B \) are disjoint then \( n(A \cup B) = n(A) + n(B) \)
2. If \( A \) and \( B \) are not disjoint then \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)

**Example 2:** Let \( G = \{1,2,3,4,5\} \), \( H = \{2,4,6\} \), \( I = \{7,8,9\} \) find

1. \( n(G \cup H) = n(G) + n(H) - n(G \cap H) = 5 + 3 - 2 = 6 \)
2. \( n(H \cup I) = n(H) + n(I) - n(H \cap I) = 3 + 3 - 0 = 6 \)
3. \( n(G \cap H) = n(G) + n(H) - n(G \cup H) = 5 + 3 - 6 = 2 \)

**Example 3:** Let \( A \) and \( B \) be subsets of a universal set \( U \). Given that \( n(B) = 9 \), \( n(A \cap B) = 5 \), and \( n(A \cup B) = 20 \) find \( n(A) \).

\[
\begin{align*}
n(A) + n(B) - n(A \cap B) &= n(A \cup B) \\
n(A) + 9 - 5 &= 20 \\
n(A) &= 16
\end{align*}
\]
Example 4: Let $A$ and $B$ be subsets of a universal set $U$. Given that $n(U) = 100$, $n(A) = 61$, $n(B) = 56$, and $n(A \cup B)^c = 30$. Find $n(A^c \cap B^c)$.

\[
\begin{align*}
\text{I} + \text{III} + \text{IV} &= 100 \\
30 + \text{II} + \text{III} + \text{IV} &= 100 \\
\text{II} + \text{III} + \text{IV} &= 70 \\
61 + \text{IV} &= 70 \\
\text{IV} &= 9 \\
\text{II} + \text{III} &= 61 \\
\text{II} + \text{IV} &= 61 \\
\text{II} &= 14 \\
\text{I}, \text{II}, \text{III} \Rightarrow 30 + 14 + 14 &= 60
\end{align*}
\]

\[
\cap (A^c \cap B^c) = \cap (A \cup B^c) = \{\text{I}, \text{III}^c\} \cup \{\text{II}, \text{III}\}
\]

Example 5: Let $n(U) = 76$, $n(A) = 45$, $n(B) = 40$, $n(C) = 41$, $n(A \cap B) = 24$, $n(B \cap C) = 22$, $n(A \cap C) = 30$, and $n(A \cap B \cap C) = 16$. Find the number in each of the following sets.

\[
\begin{align*}
\text{Find Total in Circles} & \quad 7 + 40 + 10 + 14 + 5 \\
& = 100 \\
\text{Universe} & - 100 \\
& = 60
\end{align*}
\]

\[
\begin{align*}
a. \ n[(A \cup B) \cap C] &= 14 + 10 + 6 \\
& = 30 \\
b. \ n[(B \cup C^c)] &= 7 + 10 + 16 + 10 \\
& = 53
\end{align*}
\]
Example 6: In a survey of 374 coffee drinkers it was found that 227 take sugar, 245 take cream, and 163 take both sugar and cream with their coffee. How many take sugar or cream, but not both?

\[ \text{Total in the Circles} = 625 \]

\[ \text{Universe} - 625 = 2 \]

\[ 374 - 64 - 163 = 62 \]

Example 7: In a consumer survey 67 students were asked about which subject they enjoy most. The survey revealed that 33 enjoy Math, 45 enjoy Science, 40 enjoy English, 17 enjoy Science and English only, 16 enjoy both Math and Science, 14 enjoy all three subjects, and 25 enjoy exactly two of the three subjects.

a. How many students surveyed enjoy Math and English?

\[ n(M \cap E) = 6 + 14 = 20 \]

b. How many students surveyed enjoy Science or English but not Math?

\[ (S \cup E) \cap M^c \rightarrow 12 + 17 + 3 = 32 \]

c. How many students surveyed enjoy at most one of these three subjects mentioned?

\[ 0 \text{ Subjects} \rightarrow 4 \]

\[ 1 \text{ Subject} \rightarrow 11 + 12 + 3 > 28 \]