[43] M. J. Olascoaga and G. Haller, *Forecasting sudden changes in environmental contamination patterns*, Proc. Natl. Acad. Sci. USA, 109 (2012), pp. 4738–4743.

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Iterative Methods for Linear Systems: Theory and Applications. *By Maxim A. Olshanskii and Eugene E. Tyrtyshnikov.* SIAM, Philadelphia, 2014. $85.00, xvi+247 pp., softcover. ISBN 978-1-611973-45-7.

With so many good books on iterative methods published in the 1990s and early 2000s, e.g., [1, 9, 11, 12, 14], it was surprising to see a new book on this subject, and I wondered what new material it might contain. The answer is that there is not a lot new on the basic iterative methods—conjugate gradients, GMRES, BiCG, QMR, etc., are the same as they were 10 years ago—but this book contains new material on preconditioners, Toeplitz and circulant matrices, and multigrid and domain decomposition methods, with a very nice arrangement of topics to tie the ideas together. The theoretical analysis is rigorous and makes use of a wide range of diverse techniques. The book would be difficult for someone who has little prior knowledge of iterative methods, but for those who are familiar with the basics and wish to learn more about analysis and applications, it is an excellent resource.

Chapter 1 deals with Krylov subspace methods and presents rigorous mathematical theory of convergence as well as optimality of approximations. Convergence estimates for nonnormal matrices are presented in terms of pseudospectra and the numerical range. I had hoped to see some of the more recent estimates such as those derived by Beckermann et al. (including one of the authors of this book) [2, 3, 4] and some discussion of the implications of Crouzeix's theorem or conjecture [7, 8] for the convergence of GMRES. Still, the theory that is presented is stated clearly and proved elegantly. It also might have been interesting to see some mention of the recently popularized IDR method [13, 10]. Algorithms are presented in a form that is easy to understand but not always best for computation. (For example, the unmodified Gram–Schmidt version of GMRES is given, with no discussion of how to update the QR factorization of the Hessenberg matrix.) The authors state that their book should be considered complementary to other books on iterative methods, and this is one of the areas in which other books should be consulted.

Chapter 2 deals with Toeplitz matrices and preconditioners, a topic that typically receives less attention in books on iterative methods. Simple and optimal circulant preconditioners are defined and analyzed, and the relation with the fast Fourier transform is explained. The topic of multilevel Toeplitz and circulant matrices is also addressed and leads naturally into the following chapters on multigrid and domain decomposition methods for problems arising from partial differential equations.

Chapter 3 gives a very nice introduction to multigrid methods, used either as stand-alone solvers or as preconditioners for conjugate gradient or other iterative methods. Relevant properties of finite element approximations are clearly delineated and then used to provide a clear and succinct proof of convergence (at a rate that is independent of the mesh size; i.e., with error reduction factor 1/2 at each step) for a two-grid method applied to the model problem of Poisson's equation. This is a chapter that I would recommend to students with or without a background in multigrid methods. I often recommend *A Multigrid Tutorial* by Briggs et al. [6], but while that book provides a very friendly, easy-to-read introduction to how multigrid methods work, the lack of mathematical analysis can lead to confusion. This book makes the mathematics very clear. I was especially interested in the proof (p. 113) that the 2-norm of the iteration matrix for the model problem is less than or equal to $1/2$, as proofs that I had seen previously dealt with the A-norm of the iteration matrix. After the section on the two-grid method there is a nice explanation of how two-grid theory can be used to establish convergence of the multigrid V-cycle or W-cycle.

Chapter 4 deals with space decomposition methods, such as alternating Schwarz methods, domain decomposition methods, and the additive multigrid method viewed as a space decomposition algorithm. There is discussion of the BPX (Bramble–Pasciak–Xu) [5] preconditioner and also of hierarchical basis methods [15]. The unifying theme is decomposition of the problem into different parts (domains, frequencies, etc.), and this point of view yields a unifying analysis.

The final chapter in the book discusses applications and modifications of the general theory that are often required to develop algorithms that work in practice. Problems considered include singularly perturbed partial differential equations and certain problems in fluid mechanics. There is discussion of different multigrid smoothers that may be needed in order to obtain convergence rates that are independent of the mesh size. Examples are ILU smoothers and a number of others designed for specific types of problems.

Overall, I enjoyed this book very much. There are a number of typos and occasional changes in notation (e.g., sometimes the grid size is $1/N$ and sometimes it is $1/(N+1)$) and I wish the authors would avoid the expression $\text{cond}(BA)$ for the ratio of the largest to smallest eigenvalue of the preconditioned matrix BA (or AB) in the conjugate gradient algorithm. This leads to confusion in Theorem 1.33 and the discussion afterwards, where the norm being used is not specified. Hopefully these small issues can be fixed if a second edition appears. This book demonstrates the blending that has taken place during the last 20 years or so of what were once considered somewhat separate and maybe even competing ideas—iterative methods, multigrid, and domain decomposition—into one cohesive whole. The mathematical theory is there, and there are helpful exercises at the end of each chapter to drive home the main points. It is a valuable new resource for the research community in iterative methods.

REFERENCES

[1] O. Axelsson, *Iterative Solution Methods*, Cambridge University Press, New York, 1994.

[2] B. Beckermann, *Image numérique, GMRES et polynômes de Faber*, C.R. Acad. Sci. Paris, Ser. I, 340 (2005), pp. 855–860.

[3] B. Beckermann and M. Crouzeix, *Faber Polynomials of Matrices for Nonconvex Sets*, preprint, arXiv.1310.1356, to appear in JAEN J. of Approx.

[4] B. Beckermann, S. A. Goreinov, and E. E. Tyrtyshnikov, *Some remarks on the Elman estimate for GMRES*, SIAM J. Matrix Anal. Appl., 27 (2006), pp. 772–778.

[5] J. Bramble, J. Pasciak, and J. Xu, *Parallel multilevel preconditioners*, Math. Comp., 55 (1990), pp. 1–22.

[6] W. L. Briggs, V. E. Henson, and S. F. McCormick, *A Multigrid Tutorial*, SIAM, Philadelphia, 2000.

[7] M. Crouzeix, *Bounds for analytical functions of matrices*, Integral Equations Oper. Theory, 48 (2004), pp. 461–477.

[8] M. Crouzeix, *Numerical range and functional calculus in Hilbert space*, J. Funct. Anal., 244 (2007), pp. 668–690.

[9] A. Greenbaum, *Iterative Methods for Solving Linear Systems*, SIAM, Philadelphia, 1997.

[10] M. Gutknecht, *IDR explained*, Electron. Trans. Numer. Anal., 36 (2009), pp. 126–148.

[11] W. Hackbush, *Iterative Solution of Large Sparse Systems of Equations*, Springer, New York, Berlin, 1994.

[12] Y. Saad, *Iterative Methods for Sparse Linear Systems*, 2nd ed., SIAM, Philadelphia, 2003.

[13] P. Sonneveld and M. B. Van Gijzen, *IDR(s): A family of simple and fast algorithms for solving large nonsymmetric systems of linear equations*, SIAM J. Sci. Comput., 31 (2008), pp. 1035–1062.

[14] H. van der Vorst, *Iterative Krylov Methods for Large Linear Systems*, Cambridge University Press, Cambridge, UK, 2003.

[15] H. Yserentant, *Old and new convergence proofs for multigrid methods*, Acta Numer., 1993 (1993), pp. 285–326.

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Climate Modeling for Scientists and Engineers. *By John B. Drake.* SIAM, Philadelphia, 2014. $69.00, viii+165 pp., softcover. ISBN 978-1-611973-53-2.

Earth's climate system has all the characteristics of a *complex system.* It has many components (atmosphere, oceans, cryosphere, biosphere, lithosphere), all components interact in complicated nonlinear ways, and there are many feedback loops that can be positive (reinforcing) or negative (inhibiting) depending on the state of the system. It is self-organizing and shows more emergent phenomena than we can handle. Unfortunately, controlled experiments are impossible: there is no Planet B. Since the beginning of the industrial revolution we have been engaged in an uncontrolled experiment, and the results don't look good. Climate change is a fact, and we had better gather our intellectual resources to see what might happen if we change our habits or if we do nothing.

Fortunately, climate models enable us to explore "what-if" scenarios through large-scale numerical simulations. In fact, the computer has become the laboratory of the climate scientist and so-called community climate models (CCMs) are the in-silico tools of the trade. The Intergovernmental Panel on Climate Change (IPCC) assesses the state of our knowledge of the climate system every four years or so on the basis of results obtained with a collection of "sanctioned" CCMs.

Climate models are *process models* that incorporate many of the processes which determine the dynamics of our climate system. But they cannot incorporate all the processes, either because we don't know enough about them or because they play out on scales that cannot be captured. As a consequence, projections generated by CCMs for the future are always subject to uncertainty. This makes climate modeling an interesting topic of research for the SIAM community.

CCMs are essentially algorithmic implementations of the laws of nature. These laws determine how the system evolves in time and are commonly formulated as (ordinary or partial) differential equations. The equations are discretized according to any of a number of approximation procedures and implemented in numerical algorithms designed for the particular computer architecture on which the simulations are to be performed. All this is the bread and butter of computational science.

The book under review was written from the perspective of a particular climate model, namely, the Community Climate System Model (CCSM), which was (and is still being) developed by a research community sponsored by the U.S. Department of Energy. The book gives an overview of what goes into the CCSM and how the algorithms are formulated and implemented. The focus is on the atmosphere and ocean, the two primary components of the climate system.

The book opens with a chapter on climate data and the basic circulation patterns of Earth's atmosphere and oceans. Chapter 2 is devoted to the formulation of the basic equations, the conservation equations of mass, momentum, and energy for a fluid on the surface of a rotating sphere. The Coriolis effect plays an important role and, depending on the scale of interest, various simplifications are possible (geostrophic wind approximation, hydrostatic approximation, shallow water approximation, etc.). The chapter ends with two examples: the hydrostatic baroclinic equations used in the atmospheric component of the Community Atmosphere Model and the continuity, momentum, and hydrostatic equations for the Parallel Ocean Program.

Chapter 3 is devoted to the discretization of the basic equations, in both the spatial domain and the time domain. The author describes in detail the control-volume method, the semi-Lagrangian transport method, and Galerkin spectral methods. In the course of the discussion, the reader learns about various solution methods (factorization, conjugate gradient, GMRES),