In a complex consisting of square rooms we hide a piece of cheese and a piece of salmon. The complex has rectangular floor plan with length and width equal to 5ft per dimension. Each room has wall of size equal to 1ft. Each wall but the exterior ones of any of the rooms has an opening wide enough for a cat and for a mouse to go through. Each room is identified by coordinates \( m, n \). We set \( m = n = 0 \) for the room in the southeast corner of the complex. The room where the salmon and the cheese are hidden has coordinates \((3, 3)\) but the animals don’t know which this room is. They are both guided to this room by following the smell of the delicacy they are going after. The cat mouse enter the complex at rooms \((m_1, n_1)\) and \((m_2, n_2)\) respectively. There are several configurations we can choose for the entry of each animal. For this problem we choose \( n_1 = n_2 = 1 \) and \( m_1 = 1 \) and \( m_2 = 4 \).

Both animals can move from the room they are only to an adjacent room through the wall openings and they can move one room per each step only, but both animals will never move backward. At time \( t = 0 \) (time assumes discrete values only, say 0, 1, 2, \ldots) each animal is at the room where each one entered the complex. Let \( X^{(c)}_1 \) and \( Y^{(c)}_1 \) be the coordinates of the room the cat will be at time step \( t = 1 \) and let \( X^{(m)}_1 \) and \( Y^{(m)}_1 \) be the coordinates of the room the mouse will be at the same time step. We assume that the cat and the mouse move independently of one another.

(a) Propose a plausible probability model, that is a joint pmf for \( X^{(c)}_1 \) and \( Y^{(c)}_1 \) and a joint pmf for \( X^{(m)}_1 \) and \( Y^{(m)}_1 \). Take into account that a move to the direction from which the sought delicacy comes from is more likely than a move to the opposite direction, but all allowable moves must be allowed a positive probability value.

(b) If, at time \( t = 1 \), the \( X \)-coordinate of the mouse is equal to 3, what are the pmfs of \( Y^{(m)}_1 \) and \( Y^{(c)}_1 \)?

(c) Using the law of total probability find the joint pmf for \( X^{(c)}_2 \) and \( Y^{(c)}_2 \) and for \( X^{(m)}_2 \) and \( Y^{(m)}_2 \).

(d) What is the probability that at time step \( t = 2 \) both animals enter the same room?

(e) What is the probability that at time step \( t = 2 \) both animals occupy adjacent rooms?