Towards Optimal Design of Aortic Reconstruction Minimizing Viscous Energy Loss

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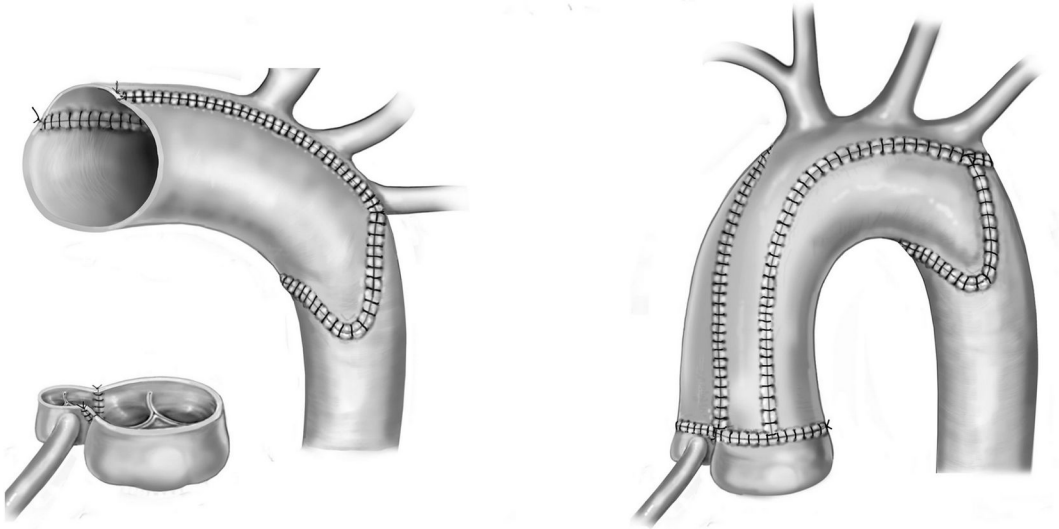
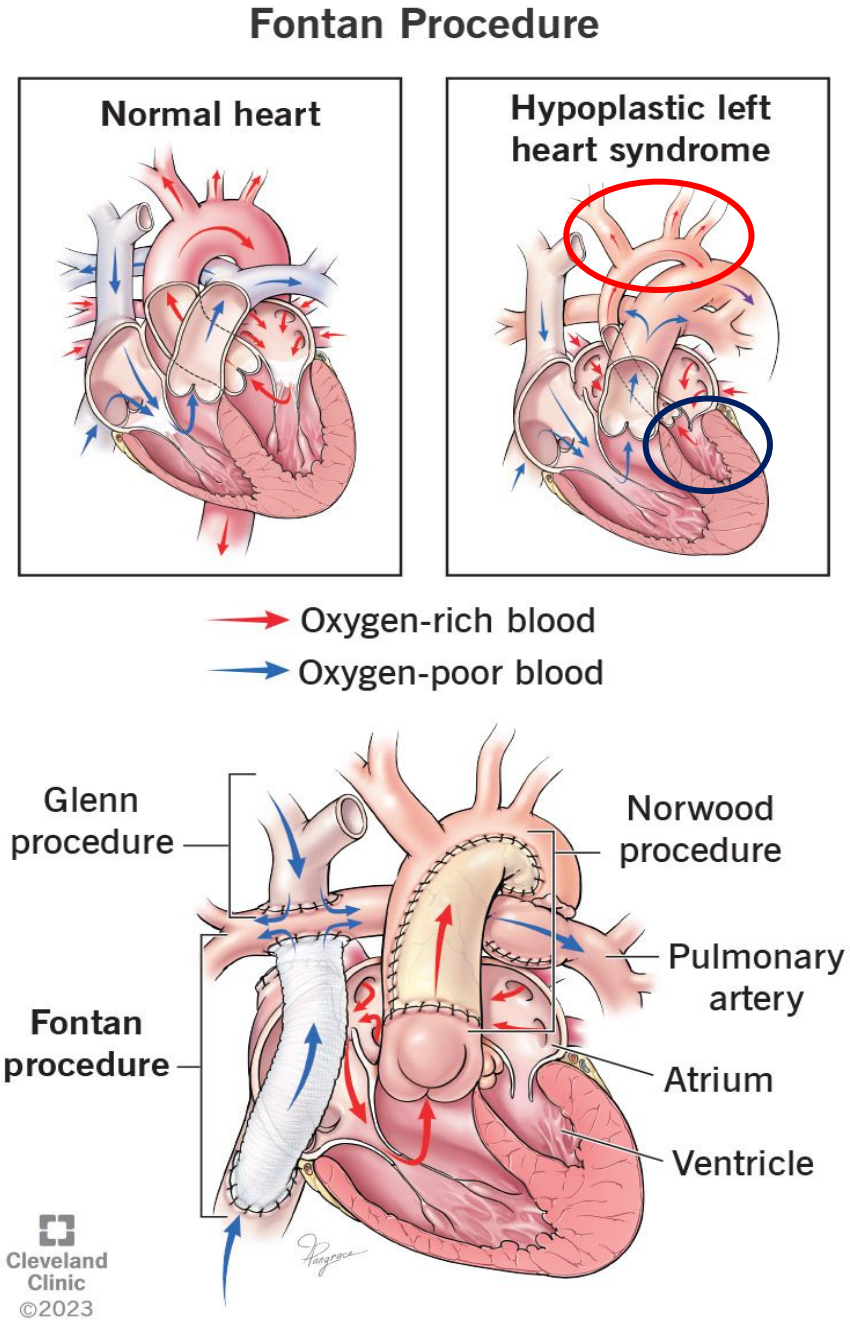
Normal Blood Circulatio

- Two ventricles pump blood between the lungs and the other organs of the body.
- In particular, the left ventricle pumps oxygenated blood to the body via the aorta.

Single Ventricle Circulation

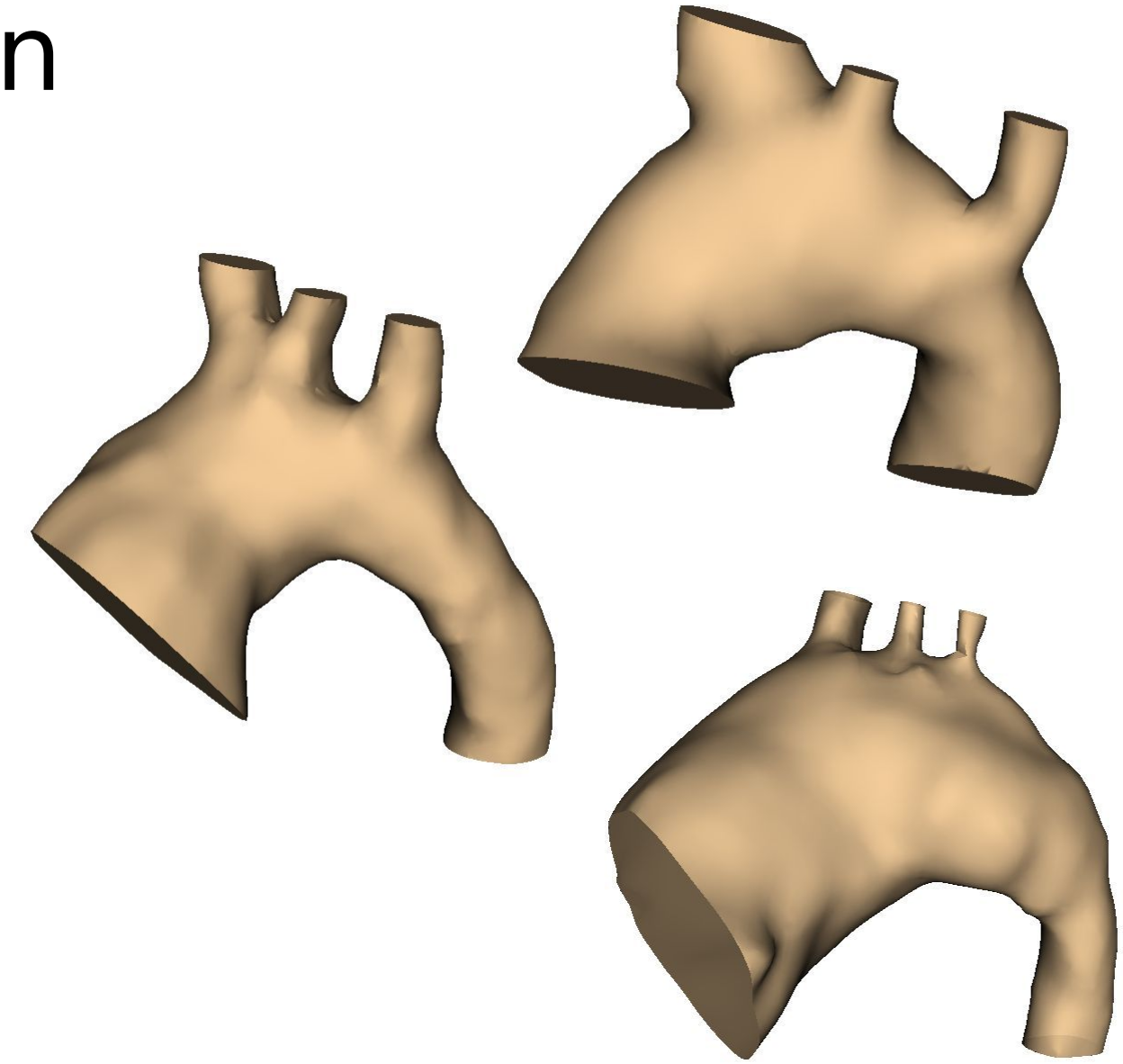
- At birth, left ventricle is “bad”
- Treatment: A series of surgeries performed within the first few years after birth “reroute” major blood vessels to accommodate a s





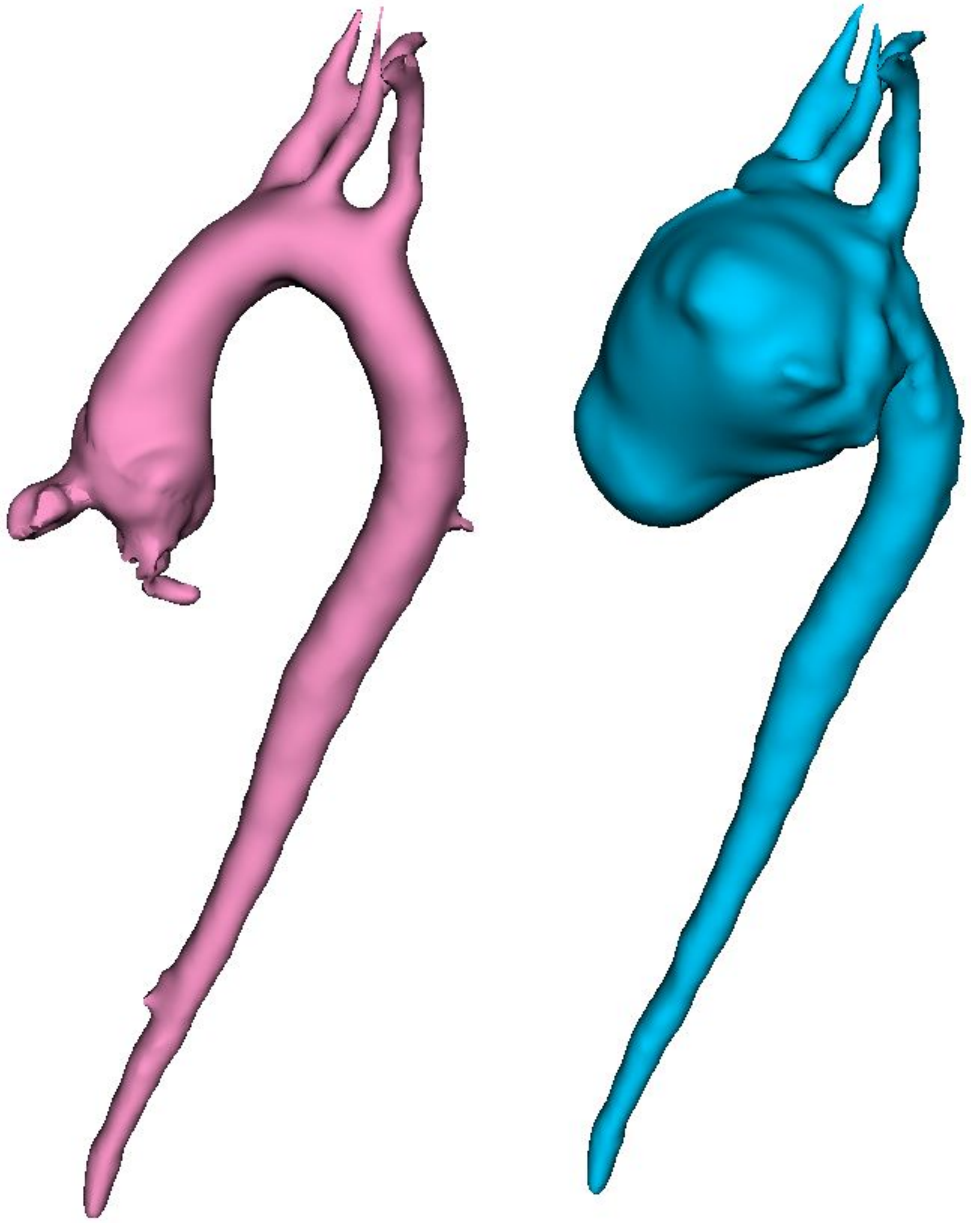
Aortic arch dilation

- Dynamic material properties of grafted wall tissues (affected by pressure and shear stress)
- Abnormal flow patterns of single ventricle circulation
- Early stage of development at which

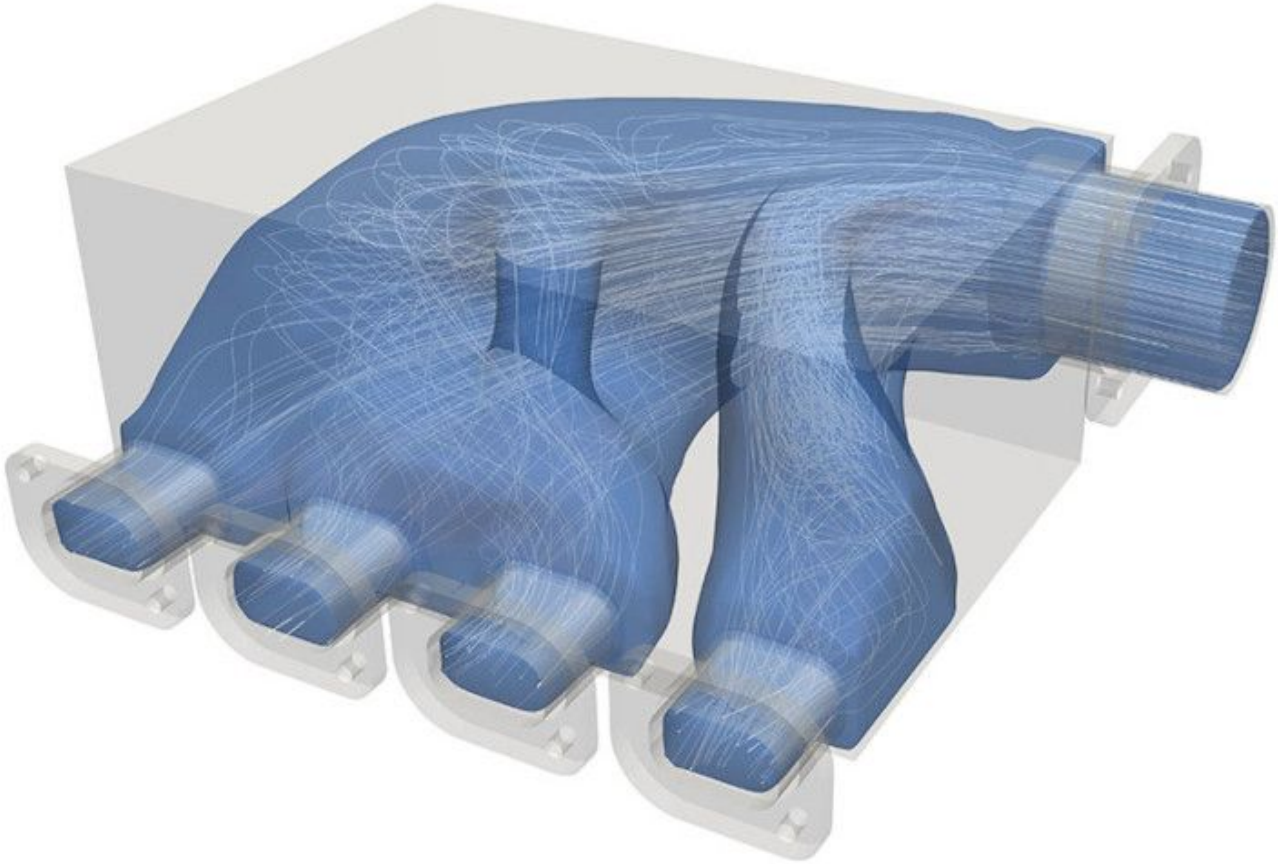


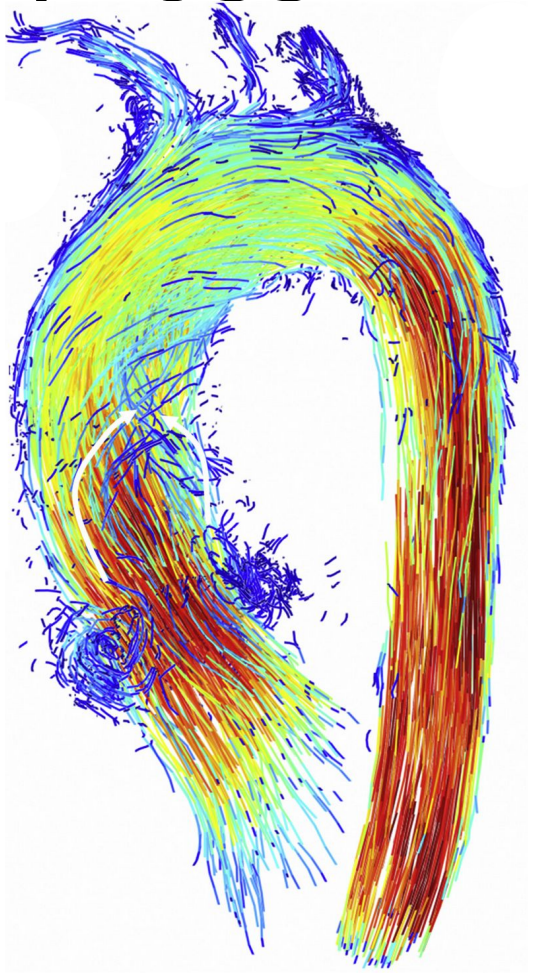
Background and Motiv

- (Intuition) Dilated aorta is less “efficient” than the normal one at blood profusion
- There is an opportunity for shape reconstruction (by grafts or girdles) at a later date
- Here, pink aorta is the result of a virtual surgery performed by a specialist on the blue aorta.
- Note: the surgeon has no quantitative information about flow patterns for this aorta.



Goal: Design an aortic arch which, subject to appropriate boundary conditions, minimizes energy loss.





Viscous Energy Loss

Assuming (for now) that the aortic wall is rigid, energy losses are accounted entirely by viscous friction:

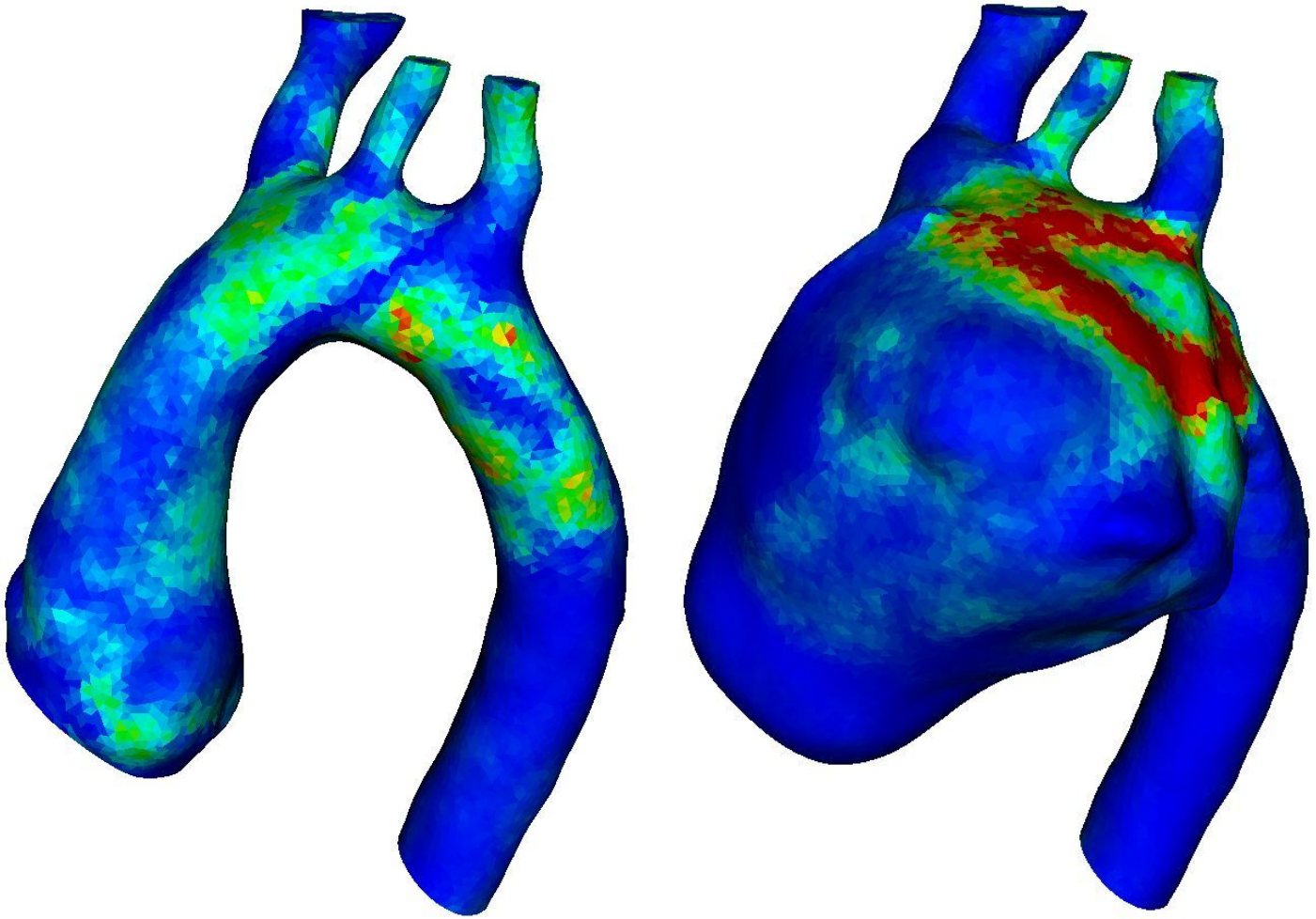
$$\phi = 2\left\|\frac{\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T}{2}\right\|^2$$

$$\frac{d}{dt}E_{viscous} = \mu \iiint \phi \, dV$$

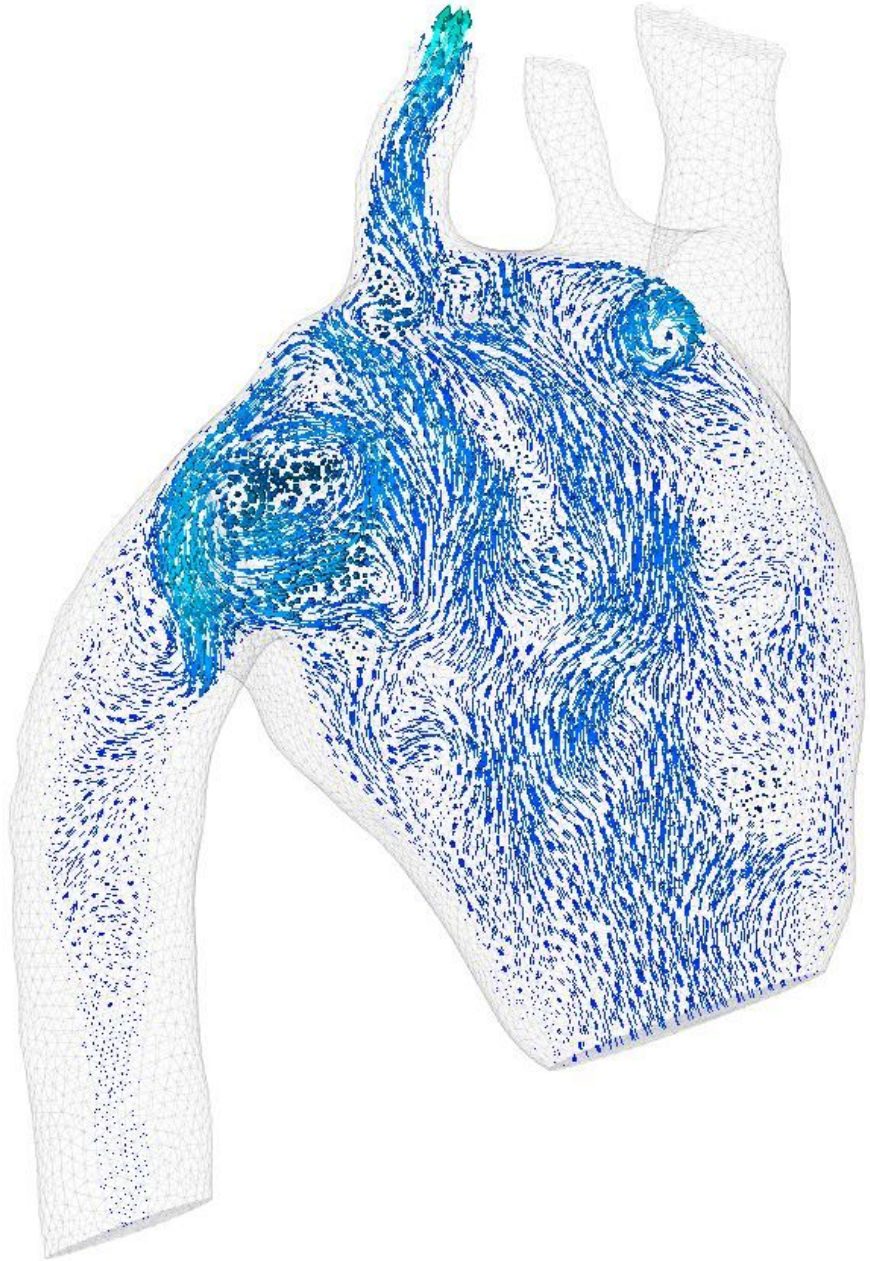
$$\frac{d}{dt}E_{viscous} \approx \mu \sum_i \phi(\Delta_i)\,|\Delta_i|$$

Energy loss rate due to viscous friction

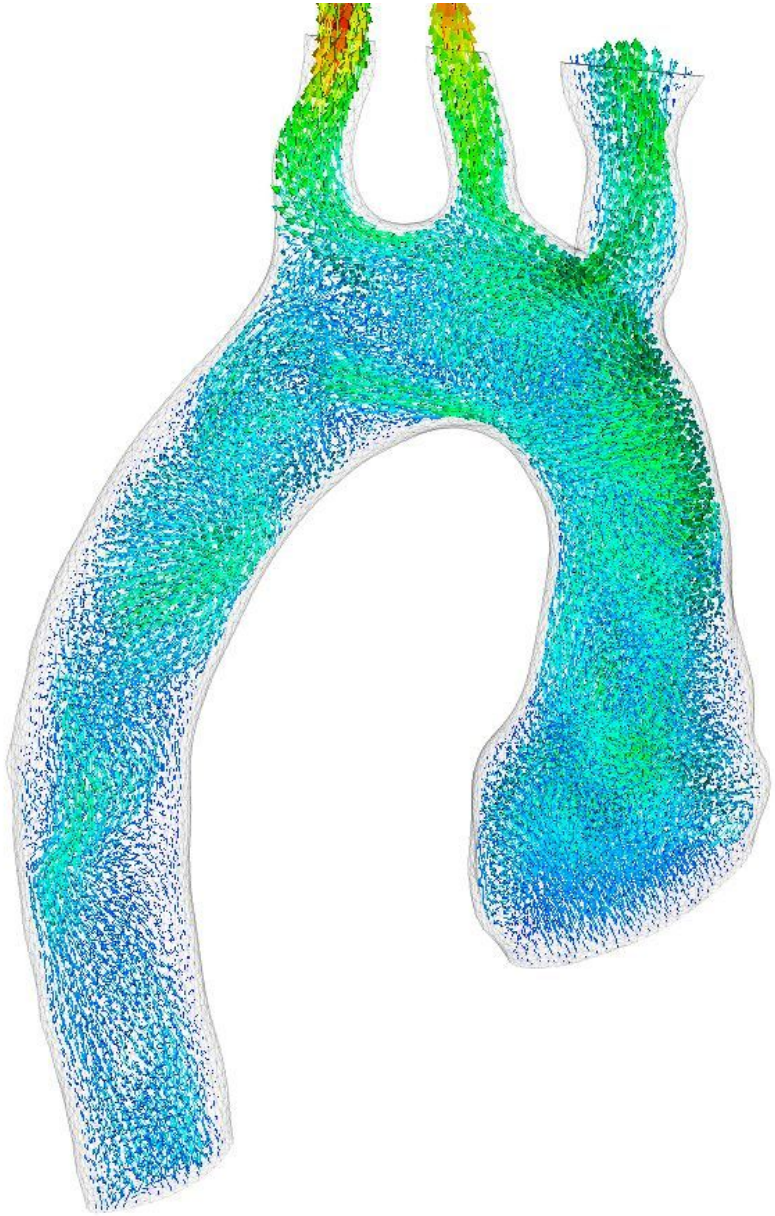
- Same geometry at the inlet and outlets.
- Outlets all have same (resistance) boundary condition
- Inlets both have the same flow waveform and velocity profile.



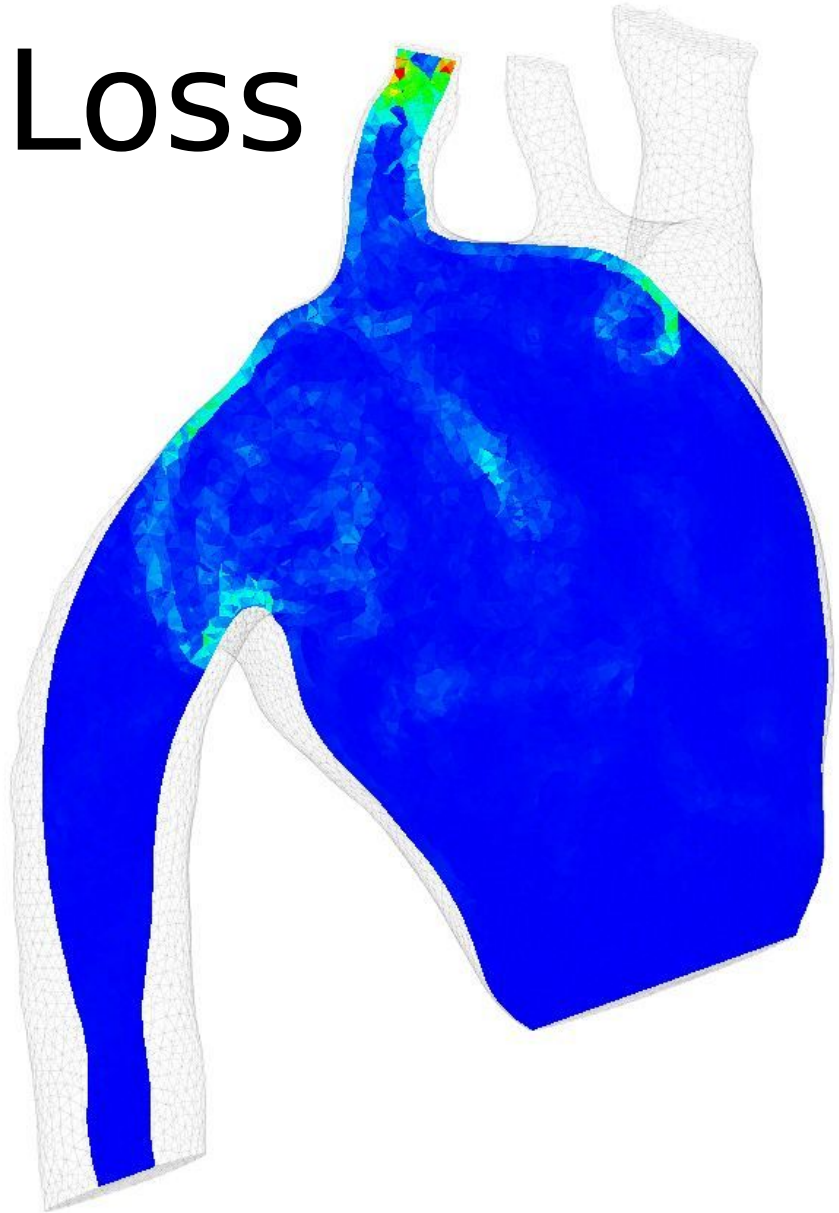
Velocity



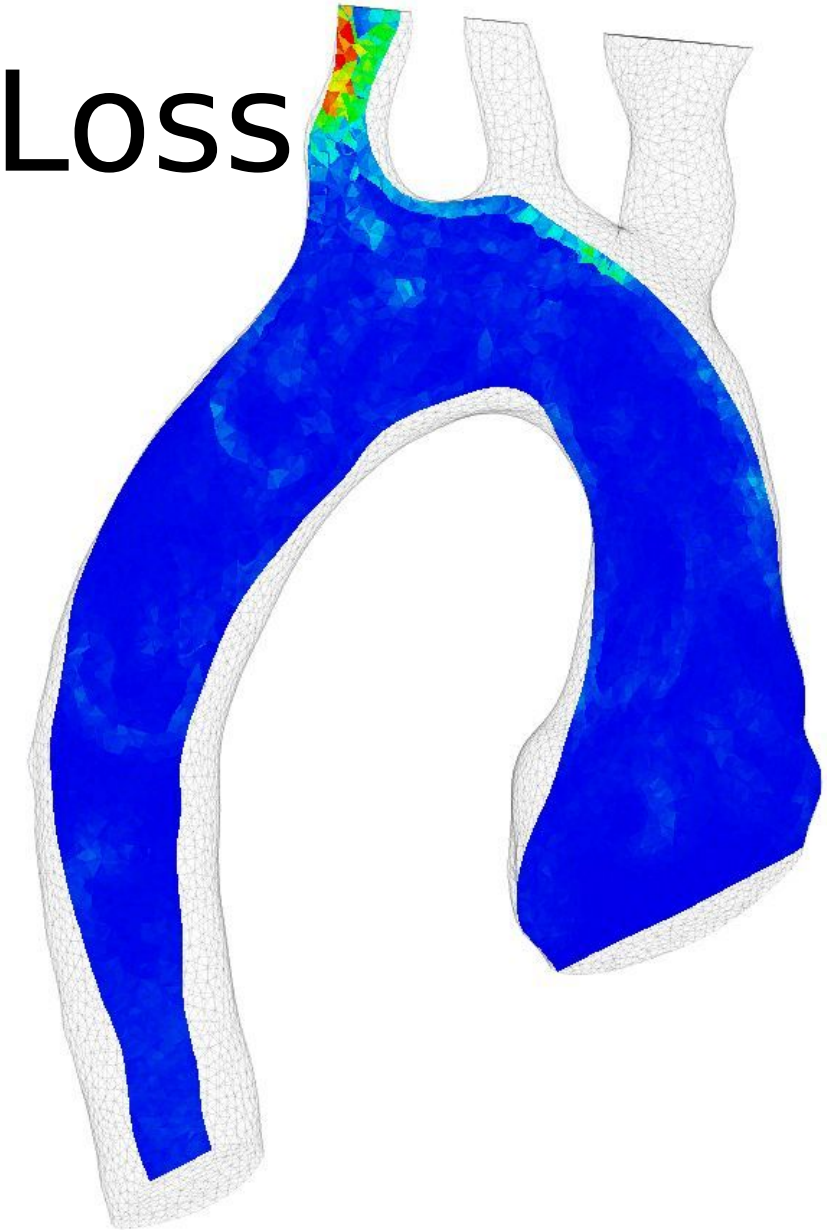
Velocity



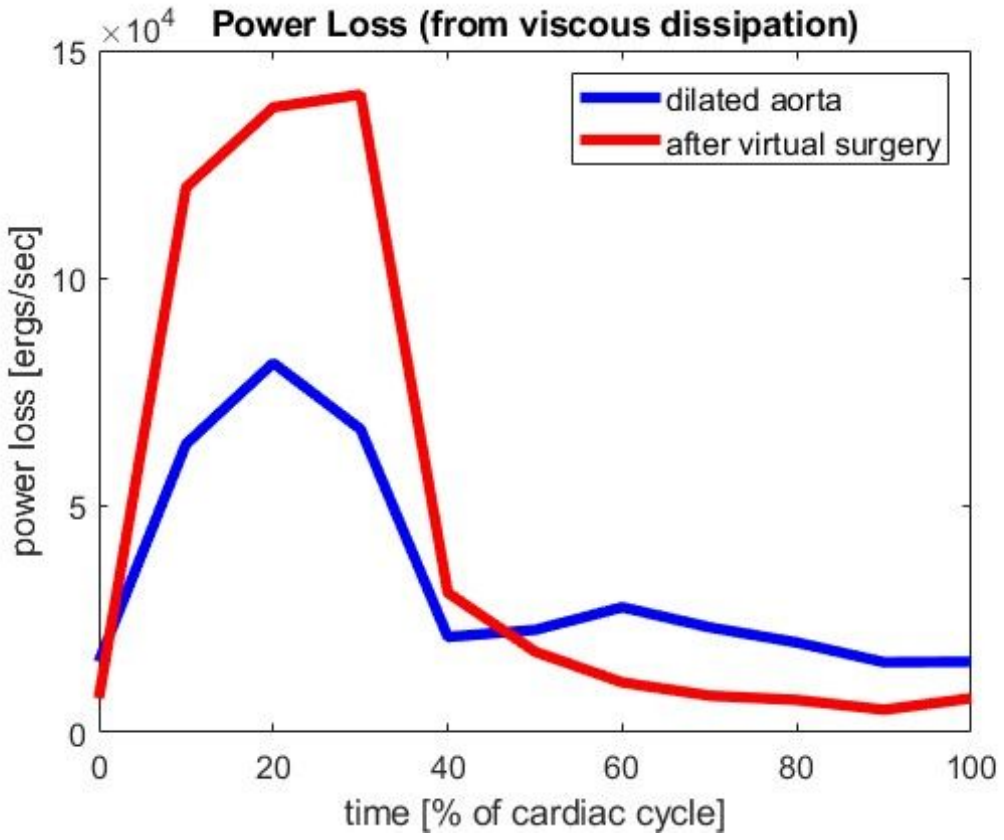
Viscous Loss

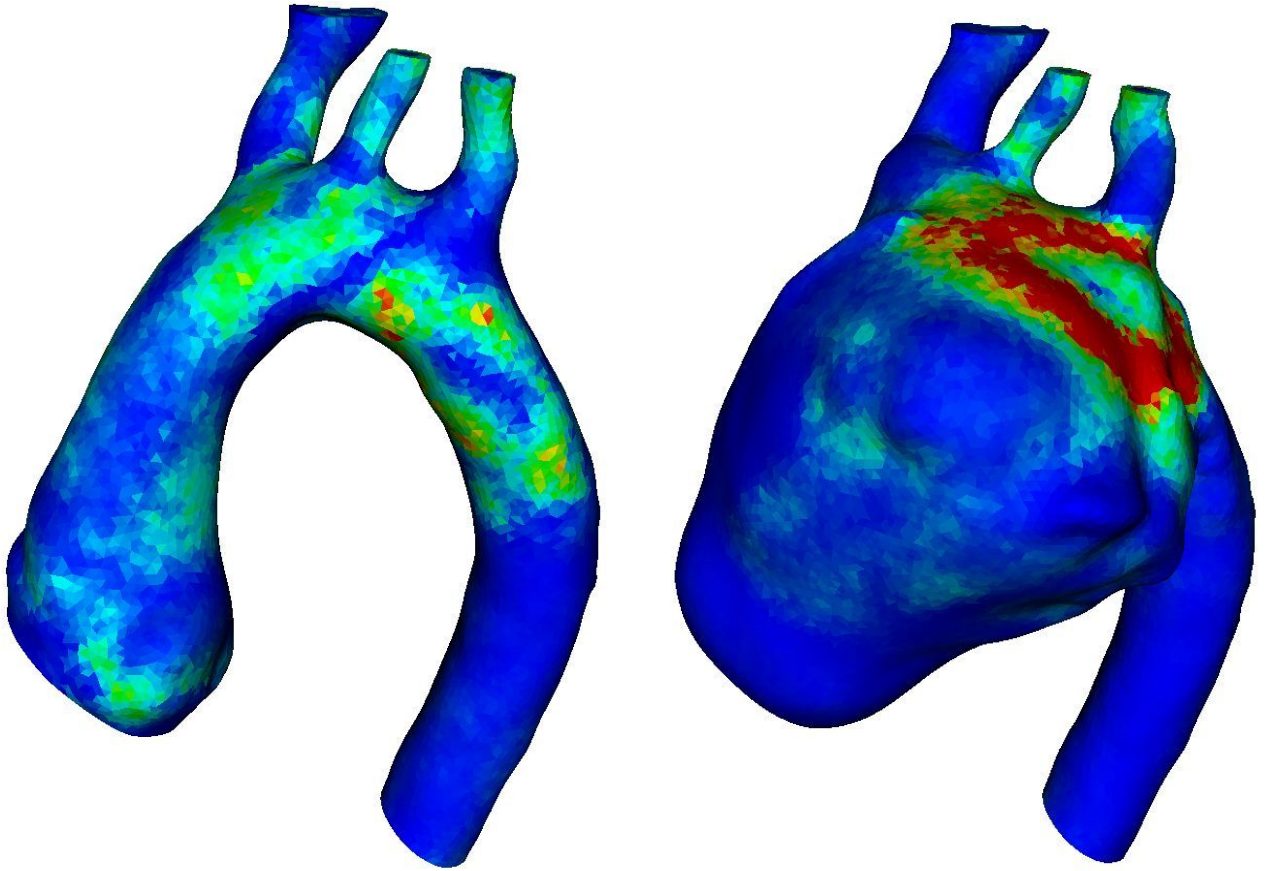


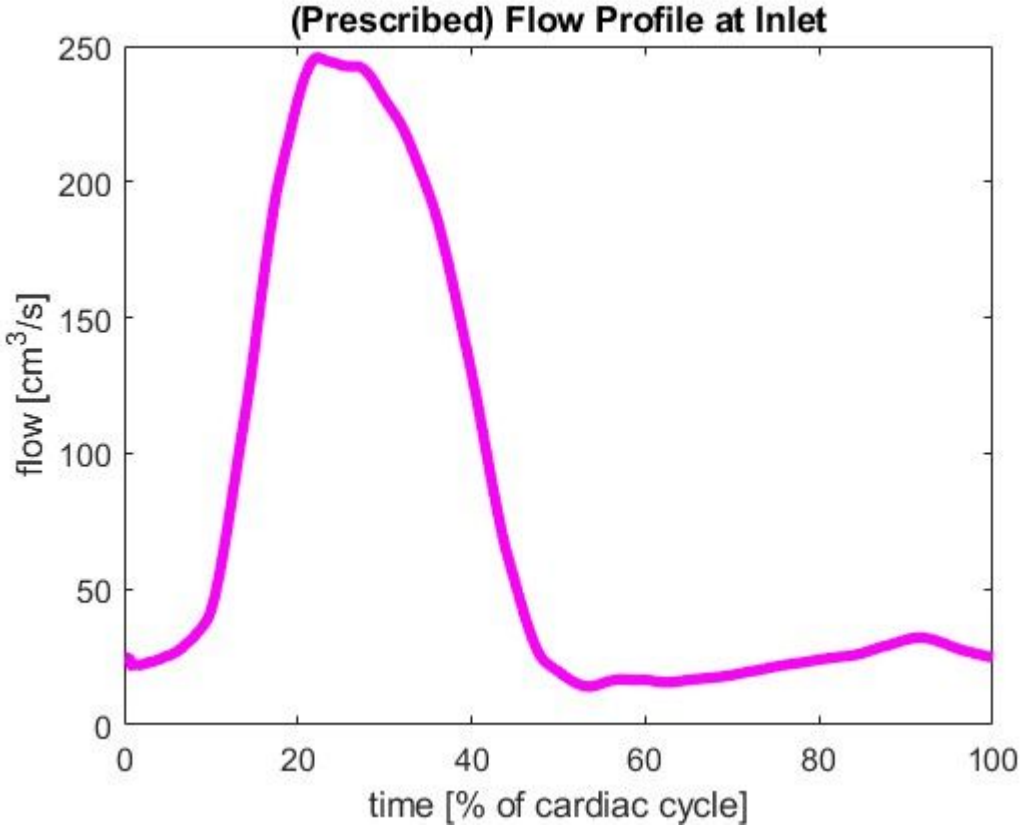
Viscous Loss

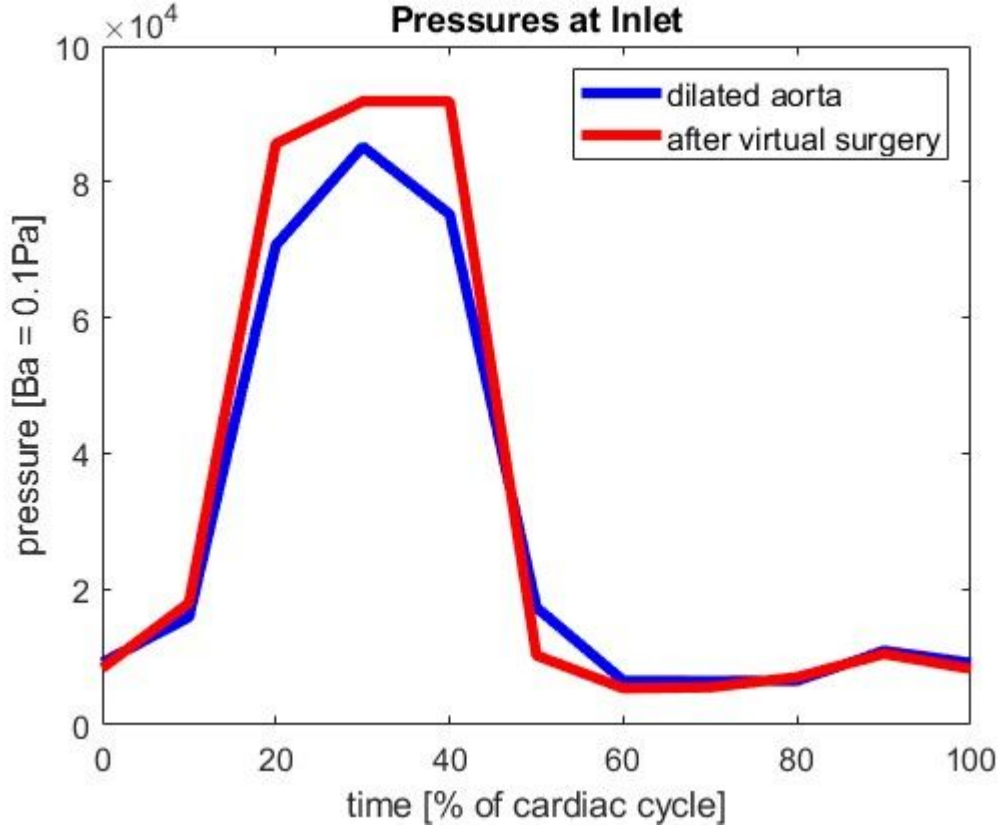


An apparently counterintuitive result?









$$\frac{d}{dt} E_{pressure} = pQ$$

Next Steps:

- Shape optimization/topology optimization for the rigid case.
- Relax the rigid model assumption.
- Account for nonuniform material properties.
- Account for surgical constraints of the reconstruction operation.