Honors Calculus, Math 1450- HW 2 (due Thursday 9th September)

Dr Matthew Nicol, PGH 665

(1) Suppose that a sphere has radius \( r(t) \) and \( \frac{dr}{dt} = r^{1/3} \). Find the rate of change with respect to time when \( r = 2 \) of the : (a) volume of the sphere; (b) surface area of the sphere.

(2) Show that \( \frac{d}{dx} \csc x = -\csc x \cot x \)

(3) Suppose, instead of measuring an angle \( \theta \) in radians, we measure \( \theta \) in degrees, where \( 2\pi \) radians equals 360 degrees. Show that

\[
\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \frac{\pi}{180}
\]

(4) Find \( \frac{dw}{dt} \) if

(i) \( w = \tan x \) and \( x = 2t^2 + 1 \)

(ii) \( w = 2^x \) and \( x = \sin(\sqrt{t}) \)

(5) Suppose \( f(x) \) is a one-to-one differentiable function and its inverse function \( f^{-1} \) is also differentiable with \( f'(x) \neq 0 \) for any \( x \). Show that

\[
(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}
\]

(6) The number \( a \) is called a double root of the polynomial function \( f \) if \( f(x) = (x - a)^2 g(x) \) for some polynomial function \( g \). Show that \( a \) is a double root of \( f \) if and only if \( a \) is a root of both \( f'(x) \) and \( f(x) \).
(6) Find \( \lim_{\theta \to 0} \frac{\sin(7\theta) - \sin(2\theta)}{\sin(2\theta)} \)

(7) Differentiate i.e. find \( f'(x) \) where

(i) \( f(x) = \ln(\ln(x^2 + 1)) \)
(ii) \( f(x) = e^{x^2}\tan(x) \)
(iii) \( f(x) = x^x \)

(8) Show that \( f(x) = x^{3/2} \) is differentiable at 0 but not twice differentiable at \( x = 0 \).

(9) Differentiate

(i) \( f(x) = \ln(\ln(x^4 + 1)) \)
(ii) \( f(x) = e^{x^2}\sin(x) \)
(iii) \( f(x) = \cot^2(x) \)

(10) Differentiate

(i) \( f(x) = \ln(\sec x + \tan x) \)
(ii) \( g(t) = \frac{1}{t^2+6} \)

(11) Find \( \frac{d^2y}{dx^2} \) if \( x \) and \( y \) are related by the equation

(i) \( x^3 + y^3 = 1 \)
(ii) \( y + \sin(y) = x \)

(12) Find the equation of the tangent line to \( x^2y - 5xy^2 = -6 \) at the point (3, 1).
(13) Section 3.5: Question 58.

(14) Use logarithmic differentiation to find the derivative of \( y = (2x + 1)^3(x^4 - 2)^5. \)

(15) The definition of \(|x|\) is

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0; \\
  -x & \text{if } x < 0.
\end{cases}
\]

Show that \( f'(x) = \frac{1}{x} \) if \( x \neq 0 \) and \( f(x) = \ln(|x|). \)