Honors Calculus, Exam 1

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Please write your answers clearly and in a logical and well-organized way. Points will be deducted for sloppy work.

Show all working.

Good Luck.
(1) [5] Where does the plane \(2x + 4y - z = 5\) intersect the line \(x = 4 + 3t, y = 3, z = -2t\)?

(2) [5] Find a unit vector orthogonal to the points \((1, -2, 0)\) and \((1, 0, 3)\).

(3) [8] Find the equation of the tangent plane to the surface \(z = f(x, y) = e^x \cos(x + y)\) at the point \((0, \frac{\pi}{3})\). Use a tangent plane approximation to estimate \(f\left(\frac{1}{10}, \frac{\pi}{3}\right)\).

(4) [5] Find the distance of the origin to the line given parametrically as \(x = 1 + 2t, y = 2 - t\) and \(z = t + 1\).

(5) [5] For a perfect gas we have the equation \(PV = RT\) where \(P\) is pressure, \(V\) is volume, \(T\) is temperature and \(R\) is a constant. If \(T\) changes from 400 to 402 degrees and \(V\) from 5 to 5.05 \(m^3\) find the approximate change in \(P\) via linearization. Note that the answer will involve the constant \(R\).

(6) [7] Show that \(u(x, t) = \sin(kx) \sin(\omega t)\) is a solution of the wave equation \(\frac{\partial^2 u}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 u}{\partial t^2}\) if \(k = \frac{\omega}{\nu}\).

(7) [5] If \(x = r \cos \theta, y = r \sin \theta\) and \(z = f(x, y)\) find:
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\frac{\partial^2 z}{\partial r^2}
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