Probability and Statistics I
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Exercise Sheet 1: due 5pm Tuesday 7th September (10 points)

(1) 4 awards are to be presented to students from a class of 10. How many different outcomes are possible if each student can

(a) receive at most 1 reward?
(b) any number of awards?
(c) at most 2 awards?

(2) If 10 people are to be divided into 3 distinct groups - one administrative, one financial and one clerical- of respective sizes 2, 3 and 5 how many divisions are possible?

(3) How many vectors \( v = (x_1, x_2, \ldots, x_k) \) are there such that each \( x_i \) is a positive integer \( 1 \leq x_i \leq n \) and \( x_1 < x_2 < \ldots < x_n \)?

(4) The partial derivatives of order \( r \) of an analytic function \( f(x_1, x_2, \ldots, x_n) \) of \( n \) variables do not depend upon the order of differentiation. Give an expression for the number of derivatives of \( r \)-th order that a function of \( n \) variables has and hence determine the number of derivatives of 4th order that a function of 3 variables has.

(5) A fair coin is tossed until the first time that the same side appears twice in succession. Let \( N \) be the number of tosses required. Determine the probability mass function, \( p(N) \), and determine the probability that \( N \) is even.
(6) Determine the mean for the probability mass function

\[ p(k) = \frac{2(n - k)}{n(n - 1)} \]

for \( k = 1, 2, 3, \ldots \).

(7) 4 independent random variables, each uniformly distributed over the interval \([0, 1]\), are added and 6 is subtracted from the total. Determine the mean and variance of the resulting random variable.

(8) Give an example of two random variables \( X, Y \) on a probability space \((\Omega, P)\) with \( E[X] \neq 0, E[Y] \neq 0, E[XY] = E[X]E[Y] \) but \( X \) and \( Y \) are not independent.

(9) Suppose \( X \) is a random variable with density function \( f(x) = kx^{k-1} \) for \( 0 \leq x \leq 1 \) \((f(x) = 0 \text{ elsewhere})\) where \( k > 0 \) is a fixed parameter. Determine

(a) the distribution function for \( X \)
(b) the mean \( E[X] \)
(c) the variance \( Var[X] \)

(10) Let \( \Omega = [0, 1] \) with \( P = dx \). Give an example of two identically distributed random variables \( X, Y \) on \((\Omega, P)\) with \( P(X \neq Y) = 1 \).