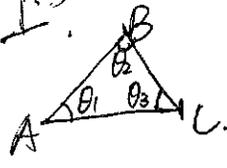


Q.10) A(2, -1, 1), B(1, -3, -5), C(3, -4, -4)



$$\vec{AB} = (-1, -2, -6)$$

$$\vec{AC} = (1, -3, -5)$$

$$\cos \theta_1 = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{-1 + 6 + 30}{\sqrt{1+4+36} \sqrt{1+9+25}} = \frac{35}{\sqrt{41} \cdot \sqrt{35}} = \frac{\sqrt{35}}{\sqrt{41}}$$

$$\theta_1 = \cos^{-1} \left(\frac{\sqrt{35}}{\sqrt{41}} \right) = 22.4915^\circ$$

$$\sin \theta_1 = \frac{\sqrt{6}}{\sqrt{41}}$$

$$\vec{BA} = (1, 2, 6)$$

$$\vec{BC} = (2, -1, 1)$$

$$\cos \theta_2 = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{2 - 2 + 6}{\sqrt{1+4+36} \sqrt{4+1+1}} = \sqrt{\frac{6}{41}}$$

$$\theta_2 = \cos^{-1} \left(\sqrt{\frac{6}{41}} \right) = 67.5085^\circ$$

$$\sin \theta_2 = \frac{\sqrt{35}}{\sqrt{41}}$$

$$\theta_3 = \pi - \theta_1 - \theta_2$$

$$\cos \theta_3 = \cos(\pi - \theta_1 - \theta_2) = -\cos(\theta_1 + \theta_2) = -(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = -\left(\frac{\sqrt{35}}{\sqrt{41}} \frac{\sqrt{6}}{\sqrt{41}} - \frac{\sqrt{6}}{\sqrt{41}} \frac{\sqrt{35}}{\sqrt{41}} \right) = 0$$

$$\theta_3 = \frac{\pi}{2} / 90^\circ$$

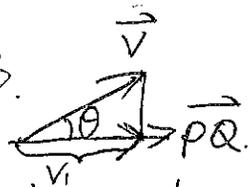
Or: $\cos \theta_3 = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| |\vec{CB}|} = 0 \Rightarrow \theta_3 = 90^\circ$

One Angle is right
Two Angles are right
Three Angles are right : 10'

$$24^{(a)} P(3, 0, 1), Q(4, 4, 4).$$

$$\vec{PQ} = (1, 4, 3) = i + 4j + 3k \quad 3.$$

$$\vec{v} = (5, 7, -1) = 5i + 7j - k.$$



the component of the force \vec{v} in the direction of \vec{PQ}

$$|\vec{v}| \cos \theta = |\vec{v}| \frac{\vec{v} \cdot \vec{PQ}}{|\vec{v}| |\vec{PQ}|} = \frac{\vec{v} \cdot \vec{PQ}}{|\vec{PQ}|} = \frac{|5 + 28 - 3|}{\sqrt{1+16+9}} = \frac{30}{\sqrt{26}}.$$

5.

$$\vec{v}_i = \frac{|\vec{v}| \cos \theta}{|\vec{PQ}|} \cdot \vec{PQ} = \frac{30}{\sqrt{26} \sqrt{26}} \cdot (1, 4, 3) = \frac{30}{26} (1, 4, 3).$$

2.

3(10) Prove that $\frac{|u|v + |v|u}{x}$ is orthogonal to $\frac{|u|v - |v|u}{y}$ for $\forall u, v$.

$$\text{to prove } x \perp y \Leftrightarrow \cos \theta = 0. \quad \cos \theta = \frac{x \cdot y}{|x| |y|} = 0.$$

$$\Leftrightarrow x \cdot y = 0.$$

2

$$(|u|v + |v|u) \cdot (|u|v - |v|u).$$

$$= |u|^2 v \cdot v - |u||v|v \cdot u + |v||u|u \cdot v - |v|^2 u \cdot u.$$

$$= |u|^2 v \cdot v - |v|^2 u \cdot u$$

$$= |u|^2 |v|^2 - |v|^2 |u|^2 = 0.$$

8

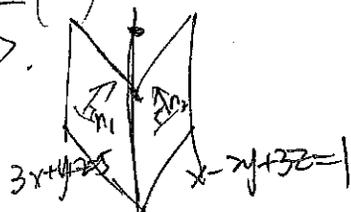
ms

4. (20). $O(0,0,0)$. $A(1,2,3)$. $B(0,-1,1)$. $C(2,0,2)$.

$\vec{n} = \vec{OA} \times \vec{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{vmatrix} = \hat{i} \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix}$
 $= 5\hat{i} - \hat{j} - \hat{k} = (5, -1, -1)$

$\vec{OC} = (2, 0, 2)$
 $\cos \theta = \frac{\vec{OC} \cdot \vec{n}}{|\vec{OC}| |\vec{n}|} = \frac{5 \times 2 + (-1) \times 0 + (-1) \times 2}{\sqrt{25+1+1} \sqrt{4+4}} = \frac{8}{\sqrt{27} \sqrt{8}} = \sqrt{\frac{8}{27}}$

$d = |\vec{OC}| \cos \theta = \sqrt{8} \times \sqrt{\frac{8}{27}} = \sqrt{\frac{64}{27}} = \frac{4}{\sqrt{27}}$

5. (20). $(4, 2, 1)$.

$\vec{n}_1 = (3, 1, 1)$ $\vec{n}_2 = (1, -2, 3)$

$\vec{v} \perp \vec{n}_1$ $\vec{v} \perp \vec{n}_2$

$\vec{v} \parallel \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = \hat{i} \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}$
 $= 5\hat{i} - 8\hat{j} - 7\hat{k}$

$= (5, -8, -7)$

$= (5, -8, -7)$

since the line passes through $(4, 2, 1)$.

$\therefore \frac{x-4}{5} = \frac{y-2}{-8} = \frac{z-1}{-7}$

scalar equation: $\frac{x-4}{5} = \frac{y-2}{-8} = \frac{z-1}{-7}$

ms

6. (20) Show $i-j$, $j-k$, $k-i$ are parallel to a plane.

$$\vec{v}_1 = i-j = (1, -1, 0)$$

$$\vec{v}_2 = j-k = (0, 1, -1)$$

$$\vec{v}_3 = k-i = (1, 0, 1)$$



If three vectors are all parallel to a plane then

the triple scalar product equals to 0.

8.

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1 - 1 = 0$$

$$\vec{v}_1 \times \vec{v}_2 \quad \vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = i \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} - j \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} + k \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = i + j + k$$

since the plane passes through the origin $(0, 0, 0)$.

$$\text{The plane is } 1(x-0) + 1(y-0) + 1(z-0) = 0$$

$$x + y + z = 0$$

7. (10) Simplify $(A \times B) \cdot (C \times D)$

Approach 2.

Approach 1.

$$\begin{aligned} & (A \times B) \cdot (C \times D) \\ &= A \times B \cdot (C \times D) \\ &= A \cdot (B \times C \times D) \\ &= A \cdot ((B \cdot D)C - D(BC)) \\ &= (A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C) \end{aligned}$$

$$\begin{aligned} & [(A \times B) \cdot (C \times D)]_i \\ &= [A \times B]_i \cdot [C \times D]_i \\ &= \epsilon_{ijk} a_j b_k \cdot \epsilon_{ilm} c_l d_m \\ &= (\delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl}) a_j b_k \cdot c_l d_m \\ &= a_j b_k c_j d_k - a_j b_k c_l d_j \\ &= [(A \cdot C)(B \cdot D) - (A \cdot D)(B \cdot C)]_i \end{aligned}$$

10.