Quiz

5 01. Find the arc length of the curve below between $(0,5,0)$ and $(1,7,3)$

$x=t.\ y=2t+5.\ z=3t.$

$R(t) = t\,i + (2t+5)\,j + (3t)\,k$ | 0 $t=0$ $(0,5,0)$

10 $R'(t) = i + 2j + 3k$ $t=1$ $(1,7,3)$

32 $s = \int_0^1 \left|\frac{dR}{dt}\right| dt = \int_0^1 \sqrt{1+2^2+3^2}\, dt = \sqrt{14}\,t\Big|_0^1 = \sqrt{14}$

5 02. A particle moves so that its coordinates at time t are given by

$x(t) = e^{-t}\cos t.\ y(t) = e^{-t}\sin t.\ z(t) = e^{-t}$

Find velocity. speed. and. acceleration.

Velocity: $v = R'(t) = x'(t)\,i + y'(t)\,j + z'(t)\,k$ 5

$= (-e^{-t}\cos t - e^{-t}\sin t)\,i + (-e^{-t}\sin t + e^{-t}\cos t)\,j + (-e^{-t})\,k.$

20 Speed. $|v| = \sqrt{(-e^{-t}\cos t - e^{-t}\sin t)^2 + (-e^{-t}\sin t + e^{-t}\cos t)^2 + (-e^{-t})^2}$ 5 10

$= \sqrt{e^{-2t}\cos^2 t + e^{-2t}\sin^2 t + \cancel{2\sin t\cos t\, e^{-2t}} + e^{-2t}\sin^2 t + e^{-2t}\cos^2 t - \cancel{2\sin t\cos t\, e^{-2t}} + e^{-2t}}$

20. $= \sqrt{3e^{-2t}} = \sqrt{3}\,e^{-t}$ 5

Acceleration: $a = R''(t)$ 5

$= x''(t)\,i + y''(t)\,j + z''(t)\,k$

$= [-(-e^{-t}\cos t - e^{-t}\sin t) - (-e^{-t}\sin t + e^{-t}\cos t)]\,i +$

$[-(-e^{-t}\sin t + e^{-t}\cos t) + (-e^{-t}\cos t - e^{-t}\sin t)]\,j + e^{-t}\,k.$ 10

$= 2e^{-t}\sin t\ i - 2e^{-t}\cos t\ j + e^{-t}\,k.$ 5