Math 6360 - Applicable Analysis

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Office hours: M,W 1-2pm or by appointment.

Prerequisites: MATH4331 or equivalent.

Texbook:

• Applied Analysis, by J. K. Hunter, Bruno Nachtergaele, World Scientific, 2005.

Supplementary Suggested Material: Functional Analysis, Sobolev spaces and Partial Differential Equations, Haim Brezis, Springer, 2011.

General description:

This course treats topics related to the solvability of various types of equations, and also of optimization and variational problems. The first half of the semester will concentrate on introductory material about norms, Banach and Hilbert spaces, etc. This will be used to obtain conditions for the solvability of linear equations, including the Fredholm alternative.

The main focus will be on the theory for equations that typically arise in applications. In the second half of the course the contraction mapping theorem and its applications will be discussed. Also, topics to be covered include finite dimensional implicit and inverse function theorems, and existence of solutions of initial value problems for ordinary differential equations and integral equations.

Tentative subjects to be covered:

- 1. Complete metric spaces, Lipschitz continuous mappings, fixed point iteration and the contraction mapping theorem.
- 2. The finite dimensional inverse and implicit function theorems
- 3. Existence-uniqueness theorems for initial value problems for systems of ordinary differential equations.
- 4. Solvability of Volterra integral equations, and representation of solutions.
- 5. Perturbation theory and the dependence of solution of equations on parameters.
- 6. Inner products, orthogonality, definitions and examples of real Hilbert spaces.
- 7. Best approximation theorem, projection theorem and Bessel's inequality.
- 8. Orthonormal bases and Parseval's equality.
- 9. Continuous linear functionals, representation theorem and dual spaces.
- 10. Continuous linear operators, adjoints and continuous bilinear forms.
- 11. Fredholm splitting theorem and the solvability of linear operator equations, the Fredholm alternative.
- 12. The Lax-Milgram theorem.
- 13. The inverse and implicit function theorem.

Grading:

Homework assigned and collected every two weeks on Monday. The homework will cover material taught in the two weeks period and will consist of 5 problems. Homework will account for 40% of your grade.

Final exam: This will be a take home project based on the material covered in the class. The project will be handed for grading in written form and presented in an oral presentation form at the end of the semester. The grade for the final exam will be based on the quality of the written and oral presentation, accuracy, correctness and style. The final exam grade will account for 60% of your final grade.