# Topology/Geometry 2013 PhD Qualifying Exam Syllabus

## 1. Topology

(1) Topological spaces and continuous maps
   (a) Open set, closed set, interior, closure
   (b) Basis, subbasis
   (c) Subspace, product, quotient, and metric topologies
   (d) Continuous map, homeomorphism
   (e) Accumulation point
(2) Nets
   (a) Directed sets, nets, subnets
   (b) Convergence of nets
   (c) Universal nets
   (d) Notions such as continuity and compactness in terms of nets
(3) Countability and separation axioms
(4) Connectedness and path connectedness
   (a) Characterization of connectedness in terms of maps into discrete spaces
   (b) Local connectedness
   (c) Path components, components, and quasi-components
(5) Compactness
   (a) Sequential compactness
   (b) Local compactness and the one-point compactification
   (c) Characterization of compact subsets of various topological spaces (*e.g.* $\mathbb{R}^N$, metric spaces)
   (d) Paracompactness
(6) Urysohn lemma, Tietze extension theorem
(7) Tychonov theorem
(8) Metrization theorems
(9) Fundamental group
   (a) Homotopy of paths
   (b) Covering spaces and path lifting
   (c) Induced map on fundamental group and its functorial properties
   (d) Homotopically equivalent spaces
   (e) Seifert/van Kampen theorem
   (f) Applications (*e.g.* retractions and the Brouwer fixed point theorem, Borsuk-Ulam theorem for $\mathbb{S}^2$)

## 2. Manifolds

(1) Topological manifolds
(2) Smooth structures, smooth manifolds, smooth manifolds with boundary
(3) Smooth maps
(4) Partitions of unity and consequences (*e.g.* a smooth version of the Tietze extension theorem)
(5) Tangent vectors, tangent bundle
(6) Differential of a smooth map
(7) Submanifolds and embedding theory
   (a) Maps of constant rank and their local structure

   (b) Immersions, embeddings
   (c) Immersed submanifolds, embedded submanifolds
   (d) The Sard theorem
   (e) Whitney embedding theorem, Whitney approximation theorems
(8) Submersions, smooth covering maps
(9) Vector fields
   (a) Lie bracket
   (b) Integral curves, flows
   (c) Lie derivative
   (d) Commuting vector fields
(10) Lie groups
   (a) Homomorphisms, subgroups
   (b) Equivariant maps
   (c) Lie algebras
(11) Cotangent bundle
(12) Covector fields
   (a) Differential of a $C^\infty$ function
   (b) Line integrals
   (c) Conservative covector fields
(13) Tensors
   (a) Multilinear algebra
   (b) Symmetric tensors, alternating tensors
   (c) Tensor fields on manifolds
(14) Riemannian manifolds
   (a) Riemannian metrics
   (b) Riemannian distance function
(15) Differential forms
   (a) Pullbacks
   (b) Wedge product
   (c) Exterior differentiation
   (d) Interior multiplication
   (e) Closed forms, exact forms
(16) Orientations of manifolds
(17) Integration on manifolds
   (a) Integration of differential forms
   (b) The Stokes theorem
   (c) Riemann volume form
   (d) Integration on Riemannian manifolds

## 3. References

(1) *Introduction to Smooth Manifolds (Second Edition)* by John Lee (Chapters 1–9, 11–16)
(2) *Topology (Second Edition)* by James Munkres (Chapters 2–7, 9, 11)
(3) *Topology and Geometry* by Glen Bredon