Numerical methods for nonlocal models: asymptotically compatible schemes and multiscale modeling

Abstract: Nonlocal continuum models are in general integro-differential equations in place of the conventional partial differential equations. While nonlocal models show their effectiveness in modeling a number of anomalous and singular processes in physics and material sciences, for example, the peridynamics model of fracture mechanics, they also come with increased difficulty in computation with nonlocality involved. In this talk, we will give a review of the asymptotically compatible schemes for nonlocal models with a parameter dependence. Such numerical schemes are robust under the change of the nonlocal length parameter and are suitable for multiscale simulations where nonlocal and local models are coupled. We will discuss finite difference, finite element and collocation methods for nonlocal models as well as the related open questions for each type of the numerical methods.