1. Let \( u_1 \) be given by \( u_1(x) = x^2 \) for all \( x \) and let \( u_2 \) be given by
\[
 u_2(x) = \begin{cases} 
 x^2 & \text{if } x \geq 0 \\
 -x^2 & \text{if } x < 0.
\end{cases}
\]
It follows that
(a) \( W[u_1, u_2](x) \neq 0 \) for all real numbers \( x \).
(b) \( W[u_1, u_2](x) \neq 0 \) for some real number \( x \).
(c) The pair of functions \((u_1, u_2)\) is linearly dependent over the set of real numbers.
(d) The pair of functions \((u_1, u_2)\) is linearly independent over the set of real numbers.
(e) None of the above.

2. If each of \( r_1 \) and \( r_2 \) is a complex number, \( J \) is an interval, and
\[
 u_1(x) = e^{r_1x} \quad \text{and} \quad u_2(x) = e^{r_2x}
\]
for all \( x \) in \( J \), then
\[
 W[u_1, u_2](x) =
\]
(a) \((r_2 - r_1)e^{(r_1+r_2)x}\) for all \( x \) in \( J \).
(b) \((r_2 + r_1)e^{(r_1-r_2)x}\) for all \( x \) in \( J \).
(c) \((r_1 - r_2)e^{(r_2+r_1)x}\) for all \( x \) in \( J \).
(d) \((r_1 + r_2)e^{(r_2-r_1)x}\) for all \( x \) in \( J \).
(e) None of the above.

3. Suppose that \( J \) is an interval, each of \( a \), \( b \), and \( c \) is a real number, \( a \neq 0 \), and \( L \) is given by
\[
 Ly(x) = ay''(x) + by'(x) + cy(x) \quad \text{for all } x \in J
\]
whenever \( y \) is a twice differentiable function defined on \( J \). Suppose that each of \( \alpha \) and \( \beta \) is a real number and
\[
 u(x) = e^{ax} \cos \beta x
\]
for all \( x \) in \( J \). It follows that \( Lu(x) =
\]
(a) \([a(a^2 - \beta^2) + b\alpha + c]e^{ax} \sin \beta x - [2a\alpha \beta + b\beta]e^{ax} \cos \beta x\) for all \( x \) in \( J \).
(b) \([a(a^2 - \beta^2) + b\alpha + c]e^{ax} \sin \beta x + [2a\alpha \beta + b\beta]e^{ax} \cos \beta x\) for all \( x \) in \( J \).
(c) \([a(a^2 - \beta^2) + b\alpha + c]e^{ax} \cos \beta x - [2a\alpha \beta + b\beta]e^{ax} \sin \beta x\) for all \( x \) in \( J \).
(d) \( [a(\alpha^2 - \beta^2) + b\alpha + c]e^{\alpha x} \cos \beta x + [2a\alpha \beta + b\beta]e^{\alpha x} \sin \beta x \) for all \( x \) in \( J \).

(e) None of the above.

4. Suppose that \( J \) is an interval, each of \( a, b, \) and \( c \) is a real number, \( a \neq 0 \), and \( L \) is given by

\[ Ly(x) = ay''(x) + by'(x) + cy(x) \] for all \( x \) in \( J \)

whenever \( y \) is a twice differentiable function defined on \( J \). Suppose that each of \( \alpha \) and \( \beta \) is a real number and

\[ u(x) = e^{\alpha x} \sin \beta x \] for all \( x \) in \( J \). It follows that \( Lu(x) = \)

(a) \( [a(\alpha^2 - \beta^2) + b\alpha + c]e^{\alpha x} \sin \beta x - [2a\alpha \beta + b\beta]e^{\alpha x} \cos \beta x \) for all \( x \) in \( J \).

(b) \( [a(\alpha^2 - \beta^2) + b\alpha + c]e^{\alpha x} \sin \beta x + [2a\alpha \beta + b\beta]e^{\alpha x} \cos \beta x \) for all \( x \) in \( J \).

(c) \( [a(\alpha^2 - \beta^2) + b\alpha + c]e^{\alpha x} \cos \beta x - [2a\alpha \beta + b\beta]e^{\alpha x} \sin \beta x \) for all \( x \) in \( J \).

(d) \( [a(\alpha^2 - \beta^2) + b\alpha + c]e^{\alpha x} \cos \beta x + [2a\alpha \beta + b\beta]e^{\alpha x} \sin \beta x \) for all \( x \) in \( J \).

(e) None of the above.

5. Suppose that each of \( a, b, \) and \( c \) is a real number, \( a \neq 0 \), and the polynomial \( P \) is given by

\[ P(z) = az^2 + bz + c \] whenever \( z \) is a complex number. Suppose that \( P \) has complex zeros \( \alpha + \beta i \) and \( \alpha - \beta i \) where each of \( \alpha \) and \( \beta \) is real and \( \beta \neq 0 \). It follows that

(a) \( [a(\alpha^2 - \beta^2) + b\alpha + c] > 0 \) and \( [2a\alpha \beta + b\beta] > 0 \).

(b) \( [a(\alpha^2 - \beta^2) + b\alpha + c] > 0 \) and \( [2a\alpha \beta + b\beta] < 0 \).

(c) \( [a(\alpha^2 - \beta^2) + b\alpha + c] < 0 \) and \( [2a\alpha \beta + b\beta] > 0 \).

(d) \( [a(\alpha^2 - \beta^2) + b\alpha + c] < 0 \) and \( [2a\alpha \beta + b\beta] < 0 \).

(e) None of the above.

6. Suppose that \( J \) is an interval, each of \( \alpha \) and \( \beta \) is a real number, and

\[ u_1(x) = e^{\alpha x} \cos \beta x \] and \( u_2(x) = e^{\alpha x} \sin \beta x \) for all \( x \) in \( J \). It follows that \( W[u_1, u_2](x) = \)

(a) \( \alpha e^{\beta x} \)

(b) \( \alpha e^{2\beta x} \)

(c) \( \beta e^{\alpha x} \)

(d) \( \beta e^{2\alpha x} \)

(e) None of the above.
7. Suppose that $J$ is an interval, each of $a$, $b$, and $c$ is a real number, $a \neq 0$, and $L$ is given by

$$Ly(x) = ay''(x) + by'(x) + cy(x)$$

whenever $y$ is a twice differentiable function defined on $J$. Suppose that the polynomial $P$ is given by

$$P(z) = az^2 + bz + cz$$

whenever $z$ is a complex number and that $P$ has complex zeros $\alpha + \beta i$ and $\alpha - \beta i$ where each of $\alpha$ and $\beta$ is real and $\beta \neq 0$. Let

$$y_1(x) = e^{(\alpha + \beta i)x} \text{ and } y_2(x) = e^{(\alpha - \beta i)x}$$

for all $x$ in $J$. Since

$$e^{i\theta} = \cos \theta + i \sin \theta$$

when $\theta$ is real, it follows that for all $x$ in $J$,

$$e^{\alpha x} \cos \beta x =$$

(a) $\frac{1}{2i} y_1(x) - \frac{1}{2i} y_2(x)$
(b) $\frac{1}{2i} y_1(x) + \frac{1}{2i} y_2(x)$
(c) $\frac{1}{2} y_1(x) - \frac{1}{2} y_2(x)$
(d) $\frac{1}{2} y_1(x) + \frac{1}{2} y_2(x)$
(e) None of the above.

8. Suppose that $J$ is an interval, each of $a$, $b$, and $c$ is a real number, $a \neq 0$, and $L$ is given by

$$Ly(x) = ay''(x) + by'(x) + cy(x)$$

whenever $y$ is a twice differentiable function defined on $J$. Suppose that the polynomial $P$ is given by

$$P(z) = az^2 + bz + cz$$

whenever $z$ is a complex number and that $P$ has complex zeros $\alpha + \beta i$ and $\alpha - \beta i$ where each of $\alpha$ and $\beta$ is real and $\beta \neq 0$. Let

$$y_1(x) = e^{(\alpha + \beta i)x} \text{ and } y_2(x) = e^{(\alpha - \beta i)x}$$

for all $x$ in $J$. Since

$$e^{i\theta} = \cos \theta + i \sin \theta$$

when $\theta$ is real, it follows that for all $x$ in $J$,

$$e^{\alpha x} \sin \beta x =$$

(a) $\frac{1}{2i} y_1(x) - \frac{1}{2i} y_2(x)$
9. The function \( y \) satisfies
\[ x^2 y''(x) - 3xy'(x) + 4y(x) = 0 \]
for all \( x > 0 \) only in case
\[ y(x) = \]
(a) \( c_1 e^{3x/2} \cos \sqrt{7}x/2 + c_2 e^{3x/2} \sin \sqrt{7}x/2 \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x > 0 \).
(b) \( c_1 e^{4x} + c_2 e^{-x} \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x > 0 \).
(c) \( c_1 x^2 + c_2 x^2 \ln x \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x > 0 \).
(d) \( c_1 e^{2x} + c_2 xe^{2x} \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x > 0 \).
(e) None of the above.

10. The function \( y \) satisfies
\[ y''(x) - 4y(x) = 0 \]
for all \( x \) only in case
\[ y(x) = \]
(a) \( c_1 \cos 2x + c_2 \sin 2x \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x \).
(b) \( c_1 \cosh 2x + c_2 \sinh 2x \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x \).
(c) \( c_1 \cos 16x + c_2 \sin 16x \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x \).
(d) \( c_1 e^{2ix} + c_2 e^{-2ix} \) for some pair of numbers \( c_1 \) and \( c_2 \) and all \( x \).
(e) None of the above.