1. Materials science

We will apply the algorithmic tools, developed in this project, to the shape optimization of electrorheological devices whose operational behavior strongly takes advantage of the rheological properties of such fluids. In fact, electrorheological fluids (ERF) are concentrated suspensions of small electrically polarizable particles with diameters in the range of micrometers dissolved in nonconducting silicon oils. The rheological effect is based on the fact that under the influence of an outer electric field the particles form chains along the field lines and then aggregate to form larger and larger columns. The impact on the macroscopic scale consists in a rapid change of the rheological properties which happens within a few milliseconds. Therefore, electrorheological fluids are used in all technological processes where a controlled power transmission plays a significant role. The field of applications ranges from automotive shock absorbers and actuators in hydraulic systems to tactile devices for virtual reality [2].

In this project, we will be concerned with the optimization of the shape of the walls of an ERF shock absorber (cf. Figure 1 (left)) in a vicinity of the inlet and outlet boundary of the ERF transfer ducts. A schematic diagram of such a shock absorber is shown in Figure 1 (left). The absorber contains two chambers filled with an ERF, a piston with two transfer ducts connecting the chambers, and a third gas-filled chamber separated from the others by a floating piston. The inner walls of the transfer ducts serve as electrodes and counter-electrodes, respectively. The electrodes are connected with an outer power source by a high voltage lead within the piston rod. As the piston moves, the fluid passes through the ducts from one chamber to the other.

In contrast to conventional shock absorbers, where the fluid chambers are filled with hydraulic oils, ERF shock absorbers have a much wider characteristics (damper force as a function of the velocity of the piston). Therefore, ERF shock absorbers offer the best compromise between safety and comfort for a wide spectrum of road conditions.

The performance of the shock absorber does not only depend on the applied voltage and the velocity of the piston, but also on the geometry of the device. In particular, the geometry of the inlet and outlet boundaries of the ducts play a decisive role. In extreme cases, cavitation due to high pressure variations may occur which negatively effects the damper characteristics. Therefore, given a prescribed pressure profile \( p_d \), the optimization issue is to design the geometry in such a way that pressure variations are minimized. Due to axisymmetry, the computational domain \( \Omega \) reduces to the right part of the fluid chamber. The inlet and outlet boundaries are represented by B-splines using the de Boor control points \( \alpha = (\alpha_1, ..., \alpha_m) \) as design variables (cf. Figure 1 (right)). Consequently, the computational domain depends on the choice of the design variables, i.e., \( \Omega = \Omega(\alpha) \).

In the stationary case, the fluid flow is described by the equations

\[
\begin{align*}
- \nabla \cdot \sigma(u,p) &= f \quad \text{in } \Omega(\alpha), \\
\nabla \cdot u &= 0 \quad \text{in } \Omega(\alpha)
\end{align*}
\]
along with appropriate boundary conditions. Here, \( \mathbf{u} = (u_1, u_2) \) is the velocity vector, \( \sigma \) refers to the stress tensor and \( f \) describes exterior forces acting on the fluid. The stress tensor \( \sigma \) is related to the rate of deformation tensor \( \mathbf{D}(\mathbf{u}) \) with \( (\mathbf{D}(\mathbf{u}))_{ij} := (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2 \), \( 1 \leq i, j \leq 2 \), by a constitutive equation where the electric field \( \mathbf{E} \) enters as a parameter. Most constitutive equations are of extended Bingham fluid type \([1, 6]\) and use some sort of power law dependence \([7]\).

Here, we use a constitutive equation developed in \([4]\)

\[
\sigma = -p \mathbf{I} + 2 \varphi(I(\mathbf{u}), |\mathbf{E}|, \mu(\mathbf{u}, \mathbf{E})) \mathbf{D}(\mathbf{u}) ,
\]

where \( \varphi \) is a viscosity function depending on the shear rate \( I(\mathbf{u}) \), the electric field strength \( |\mathbf{E}| \), and the angle \( \mu(\mathbf{u}, \mathbf{E}) \) between the velocity field \( \mathbf{u} \) and the electric field \( \mathbf{E} \). In particular, the viscosity function \( \varphi \) is assumed to be of the form

\[
\varphi(I(\mathbf{u}), |\mathbf{E}|, \mu(\mathbf{u}, \mathbf{E})) = b(|\mathbf{E}|, \mu(\mathbf{u}, \mathbf{E})) (\varepsilon + I(\mathbf{u}))^{-\frac{1}{2}} + \psi(I(\mathbf{u}), |\mathbf{E}|, \mu(\mathbf{u}, \mathbf{E})) ,
\]

where \( \varepsilon > 0 \) is a regularization parameter. In practice, the functions \( b \) and \( \psi \) are approximated by splines fitted to rheometrical data obtained for different electric field strengths.

The design objective is to minimize the difference between the actual pressure gradient and a given desired pressure gradient. If

\[
J(\mathbf{u}, p, \alpha) := \int_{\Omega(\alpha)} |\nabla p - \nabla p_d|^2 \, dx ,
\]
then the shape optimization problem can be stated as follows

\[
\begin{align*}
\min_{(u, p, \alpha)} & \quad J(u, p, \alpha) \\
\text{s.t.} & \quad \text{state equations (1.1),(1.2)}, \\
& \quad \alpha_i^{\text{min}} \leq \alpha_i \leq \alpha_i^{\text{max}}, \quad 1 \leq i \leq m.
\end{align*}
\]

The inequality constraints (1.5) on the design variables which are motivated by technological constraints on the shape of the inlet and outlet boundaries.

Assuming a known angle \(\mu(u, E)\) between the velocity vector and the exterior electric field, the flow model (1.1)-(1.3) will be discretized in space by \(Q_2 - P_1\) Taylor-Hood elements with respect to a simplicial triangulation of a reference computational domain \(\Omega(\hat{\alpha})\), where \(\hat{\alpha} \in \mathbb{R}^m\) is chosen within the set of feasible design parameters such that \(\Omega(\alpha) = \Phi(\Omega(\hat{\alpha}))\).

**Proposed Research.** Compared to shape optimization problems involving Newtonian fluids discussed, e.g., in [3, 5] the analytical treatment of the shape optimization problem (1.4–1.5) and its effective numerical solution pose new difficulties arising from, e.g., the presence of the pressure gradient in the objective and the nonlinearities in the constitutive equation (1.3). We propose to analyze the well-posedness of (1.4–1.5), to characterize its solutions and to study approximation properties of its discretization. We will then tailor multilevel preconditioners to this problem and integrate them into the primal-dual interior point method to be developed in this project.

**References**


