Optimization Theory
(3rd Homework Assignment)

**Exercise 5 (Simplex Method for a Production Planning Problem)**

The ACME Electronics Company makes black-and-white (bw) and color television sets. Each bw set requires 3 hours of assembly, 1 hour of finishing, and 2 hours of testing. Each color set requires 5 hours of assembly, 3 hours of finishing, and 2 hours of testing. For this week’s production run, ACME has required 3900 hours of assembly, 2100 hours of finishing, and 2200 hours of testing. If a bw set and a color set bring in $100 and $200 profit, respectively, then how many of each set should the company produce this week in order to maximize profit, and what is the maximum achievable profit?

The problem has been set up as a linear program in Exercise 2. Now, solve it by the simplex method starting from $J_0 = (3, 4, 5, 6)$. Perform two simplex steps resulting in $J_1, J_2$ and answer the following questions:

(a) Answer the following questions about $J_0$:

(i) The basis variables are ____ , and the nonbasis variables are ____ . The profit is ____ . There are available ____ assembly hours, ____ finishing hours, and ____ testing hours. ____ bw sets and ____ color sets are being produced.

(ii) If the number of bw sets produced were increased by one, then the profit would increase/decrease by ____ . If the number of color sets produced were increased by one, then the profit would increase/decrease by ____ .

(iii) Each extra color set produced uses up ____ assembly hours, ____ finishing hours, and ____ testing hours.

(iv) The company has enough assembly hours to make ____ color sets. The company has enough finishing hours to make ____ color sets. The company has enough testing hours to make ____ color sets. The company will therefore produce ____ color sets, at which point the company will have used all of their ____ , and the profit will now be ____ .

(b) Answer the following questions about $J_1$:

(i) The basis variables are ____ , and the nonbasis variables are ____ . The variable which has just entered the basis is ____ . The variable which has just left the basis is ____ . The profit is ____ . There are available ____ assembly hours, ____ finishing hours, and ____ testing hours. ____ bw sets and ____ color sets are being produced.
(ii) If the number of bw sets produced was increased by one, then the profit would increase/decrease by _____. How can any bw sets be produced at this point, if there are no finishing hours available? Why doesn’t the profit increase by $100? If one hour of finishing time were freed up, then the profit would increase/decrease by ____. How would one free up an hour of finishing time? The variable which should be increased is _____.

(iii) ACME has available ____ assembly hours. Each extra bw set produced reduces the number of available assembly hours by ____ units. Why is the answer not 3, because in the initial statement we were told that each bw set requires 3 assembly hours? ACME has enough assembly hours to make ____ bw sets.

(iv) ACME has available ____ testing hours. Each extra bw set produced reduces the number of available testing hours by ____ units. Why is the answer not 2? ACME has enough available testing hours to make ____ bw sets.

(v) ACME is making ____ color sets. Each extra bw set produced requires ACME to produce ____ fewer color sets. Why does ACME need to produce fewer color sets in order to produce an additional bw set? ACME has enough color sets to make ____ bw sets.

(vi) ACME has enough assembly hours to make ____ bw sets. The company has enough testing hours to make ____ bw sets. The company has enough color sets to produce ____ bw sets. The company will therefore produce ____ bw sets, at which point ACME will have used up all of their ____ and the profit will now be ____.

(c) Answer the following questions about $J_2$:

(i) The basis variables are ____ and the nonbasis variables are ____. The variable which has just entered the basis is ____. The variable which has just left the basis is ____. The profit is _____. There are available ____ assembly hours, ____ finishing hours, and ____ testing hours. ____ bw sets and ____ color sets are being produced.

(ii) If one assembly hour were freed up (keeping the amount of available finishing hours equal to zero), the profit would increase/decrease by ____. If one finishing hour were freed up (keeping the amount of available assembly hours equal to zero), the profit would increase/decrease by ____.

(iii) How do you know this tableau represents an optimal solution? How do you know there are no other optimal solutions?

(iv) The maximum profit of ____ is attained by producing ____ bw sets and ____ color sets. At this level of production, ACME has ____ unused assembly hours, ____ unused finishing hours, and ____ unused testing hours.

(v) Suppose ACME had an extra assembly hour to start with, i.e., 3901 instead of 3900. Then, their maximum profit would have been ____ instead of ____. They would therefore be willing to pay up to ____ for this extra assembly hour. With this extra hour, they would achieve maximum profit by producing ____ more/fewer bw sets and ____ more/fewer color sets.

15 Points
Homework may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.