Exercise 5: (Runge Kutta Fehlberg method)

A seven stage, embedded Runge Kutta Fehlberg method of type RK5(4) has been named after Dormand and Prince. It is characterized by the Butcher scheme

<table>
<thead>
<tr>
<th>0</th>
<th>( \frac{1}{5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>( \frac{3}{10} )</td>
<td>( \frac{3}{10} )</td>
</tr>
<tr>
<td>( \frac{4}{5} )</td>
<td>( \frac{44}{55} )</td>
</tr>
<tr>
<td>( \frac{8}{9} )</td>
<td>( \frac{19372}{6561} )</td>
</tr>
</tbody>
</table>

Provide a code which solves the satellite problem from Exercise 4 (4th Practical Assignment) by using the Dormand/Prince RKF5(4) method combined
with step size control and Fehlberg’s trick to reduce the number of function evaluations.
Choose \( h_0 = 10^{-4} \) as initial step size and
\[
TOL = 10^{-10}, 10^{-12}, 10^{-14}, 10^{-16}, 10^{-18}, 10^{-20}
\]
as tolerance in the step size control.
The code should provide a protocol file \( \text{prot} \) of the form
\[
\begin{align*}
t_0 & \quad x_1(t_0) & \quad x_1'(t_0) & \quad x_2(t_0) & \quad x_2'(t_0) \\
t_1 & \quad x_1(t_1) & \quad x_1'(t_1) & \quad x_2(t_1) & \quad x_2'(t_1) \\
t_2 & \quad x_1(t_2) & \quad x_1'(t_2) & \quad x_2(t_2) & \quad x_2'(t_2) \\
\vdots & \quad \vdots & \quad \vdots & \quad \vdots & \quad \vdots \\
t_N & \quad x_1(t_N) & \quad x_1'(t_N) & \quad x_2(t_N) & \quad x_2'(t_N)
\end{align*}
\]
where \( N \) denotes the total number of steps.

**Delivery:** Output of the code including comments and a graphical visualization of the Arenstorf orbit.

**Control results:**

<table>
<thead>
<tr>
<th>TOL</th>
<th>N</th>
<th>( x_1(T) )</th>
<th>( x_1'(T) )</th>
<th>( x_2(T) )</th>
<th>( x_2'(T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-10} )</td>
<td>75</td>
<td>-1.3224e-01</td>
<td>-4.0660e-01</td>
<td>-1.1211e+00</td>
<td>-1.4286e-01</td>
</tr>
<tr>
<td>( 10^{-12} )</td>
<td>118</td>
<td>-1.8399e-01</td>
<td>-4.4155e-01</td>
<td>-1.1352e+00</td>
<td>-8.2852e-02</td>
</tr>
<tr>
<td>( 10^{-14} )</td>
<td>190</td>
<td>-4.0103e-01</td>
<td>7.3138e-01</td>
<td>-5.5213e-01</td>
<td>1.2752e-01</td>
</tr>
<tr>
<td>( 10^{-16} )</td>
<td>331</td>
<td>8.8687e-01</td>
<td>5.9272e-01</td>
<td>4.1903e-02</td>
<td>6.3018e-03</td>
</tr>
<tr>
<td>( 10^{-18} )</td>
<td>524</td>
<td>8.8815e-01</td>
<td>5.9410e-01</td>
<td>4.1914e-02</td>
<td>2.8523e-03</td>
</tr>
<tr>
<td>( 10^{-20} )</td>
<td>935</td>
<td>9.9400e-01</td>
<td>-2.4169e-02</td>
<td>-1.5334e-04</td>
<td>-2.0013e+00</td>
</tr>
</tbody>
</table>

Delivery of the practical work at latest on April 13, 2006. The delivery may be submitted either electronically (rohop@math.uh.edu) or as a hardcopy in class.