Math 2318 Exam 1. Sanders Fall 2023

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Consider the matrices.

\[ A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D = (1 \ 1), \quad E = (3 \ 1) \]

Compute the following when defined.

(a) \( A + C \)  
(b) \( D + 2E \)  
(c) \( CB \)  
(d) \( DA \)

2. Write each of the following linear systems as an augmented matrix. Reduce to echelon form by Gaussian elimination. Finally determine all solutions if there is one.

(a) \[
\begin{align*}
\begin{array}{ccc}
2x_1 + 2x_2 + x_3 &= 8 \\
x_1 + x_2 + 2x_3 &= 10 \\
x_1 + x_2 + x_3 &= 6
\end{array}
\]

(b) \[
\begin{align*}
\begin{array}{ccc}
x_1 + 2x_2 + x_3 &= 2 \\
2x_1 + x_2 + 2x_3 &= 4 \\
x_1 + 2x_2 + 2x_3 &= 3 \\
2x_1 + x_2 + x_3 &= 3
\end{array}
\]

3. Answer the following.

(a) Is \( \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{pmatrix} \right\} \)?

(b) Is \( \begin{pmatrix} 4 \\ 5 \\ 4 \\ 5 \end{pmatrix} \in \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} \right\} \)?

If true, write the given vector as a linear combination of the two spanning vectors. Please show all your work.

4. Consider the following two subspaces.

(a) \( V_a = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \subseteq \mathbb{R}^3 \).

(b) \( V_b = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \subseteq \mathbb{R}^3 \).

For each subspace, determine its standard basis, and also state its dimension.

5. Consider the following two matrices, \( A \), each which defines a linear operator via matrix multiplication; i.e. \( L(x) = Ax \).

(a) \( A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \)  
(b) \( A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \)

Determine a basis for \( L \)'s null space and also compute the standard basis for \( L \)'s range space.