Sanders
2318 E2
Sp 25

1 a) Find inverse

$$A=\begin{pmatrix}2 & 1\\ 3 & 4\end{pmatrix}$$

I'll use Cramer's rule

$$A^{-1}=\frac{1}{8-3}\begin{pmatrix}4 & -1\\ -3 & 2\end{pmatrix}$$

$$=\boxed{\frac{1}{5}\begin{pmatrix}4 & -1\\ -3 & 2\end{pmatrix}}$$

b) $$A=\begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 2\\ 1 & 1 & 2\end{pmatrix}$$

Use Elimination on This one.

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 2 & 0 & 1 & 0\\ 1 & 1 & 2 & 0 & 0 & 1\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 2 & 0 & 1 & 0\\ 0 & 0 & 1 & -1 & 0 & 1\end{array}\right]$$

(over)

②

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}}$$

2 Calc det

a) $\det\begin{pmatrix} 2 & 2 & 1 \\ 3 & 4 & 2 \\ 6 & 4 & 5 \end{pmatrix}$ I'll use criss-cross

$$= [40 + 24 + 12] - [24 + 16 + 30] = \boxed{6}$$

③

b) $\det\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

I'll cofactor along col #1.

$$= 1(-1)^{1+1}\det\begin{pmatrix} 2 & 0 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

criss-cross These

$$+1(-1)^{2+1}\det\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$= 1\left\{[4+2]-[2+2]\right\}$$

$6-4$

$$-1\left\{[2+2]-[2+1]\right\}$$

$4-3$

$$= 2-1 = \boxed{1}$$

(4)

3. Use Cramer's rule to solve.

a) $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$$x_1 = \frac{\det\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}}{\det\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}} = \frac{4}{-4} = \boxed{-1}$$

$$x_2 = \frac{\det\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}}{\det\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}} = \frac{-6}{-4} = \boxed{\frac{3}{2}}$$

b) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$x_1 = \frac{\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}}{\det\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}} = \frac{-1}{[2]-[4+1]}$$ (over)

⑤

$$\boxed{x_1 = \frac{1}{3}}$$

$$x_2 = \frac{\det\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}{-3} = \frac{-\{[-1]\}}{-3} = \boxed{-\frac{1}{3}}$$

$$x_3 = \frac{\det\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{pmatrix}}{-3} = \frac{-1}{-3} = \boxed{\frac{1}{3}}$$

4 Find evals & evecs.

evals: $\det\begin{pmatrix} 3-\lambda & -1 \\ 2 & -\lambda \end{pmatrix} = \lambda(\lambda-3)+2$

$= \lambda^2 - 3\lambda + 2$

$= (\lambda-1)(\lambda-2) = 0$

$\lambda = 1$

$\lambda = 2$

$(A-\lambda I)\vec{v} = \vec{0}$

Evec for $\lambda = 1$ $\left[\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & -1 & 0 \end{array}\right]$ (over)

(6)

$$\sim \left[\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$r_2 = \alpha$

$2r_1 - \alpha = 0$

$r_1 = \alpha/2 \qquad \alpha = 2$

$$\boxed{\begin{array}{l} \lambda = 1 \\ \vec{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{array}}$$

$\lambda = 2 \qquad (A - \lambda I)\vec{r} = \vec{0}$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$r_2 = \alpha$

$r_1 = \alpha$

$\alpha = 1$ WLOG.

$$\boxed{\begin{array}{l} \lambda = 2 \\ \vec{r} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array}}$$

b) evals $\det\begin{pmatrix} 4-\lambda & 5 \\ -2 & -2-\lambda \end{pmatrix}$

$$= (\lambda - 4)(\lambda + 2) + 10 = \lambda^2 - 2\lambda + 2 = 0$$

Quad form $\lambda = \dfrac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = \boxed{1 \pm i}$

⑦

Evec $\lambda = 1-i$

$$(A - \lambda I)\vec{r} = \vec{0}$$

$$\left[\begin{array}{cc|c} 4-(1-i) & 5 & 0 \\ -2 & -2-(1-i) & 0 \end{array}\right] = \left[\begin{array}{cc|c} 3+i & 5 & 0 \\ -2 & -3+i & 0 \end{array}\right]$$

mult row 1 by $3-i$

$$\left[\begin{array}{cc|c} 10 & 5(3-i) & 0 \\ -10 & 5(-3+i) & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & (3-i) & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$r_2 = \alpha$

$$2r_1 + \alpha(3-i) = 0$$

$$r_1 = -\frac{\alpha(3-i)}{2}$$

WLOG $\alpha = 2$

$$\boxed{\lambda = 1-i \quad \vec{r} = \begin{pmatrix} -3+i \\ 2 \end{pmatrix}}$$

(8)

Evec for $\lambda = 1+i$

$$(A-\lambda I)\vec{r} = \vec{0}$$

$$\left[\begin{array}{cc|c} 4-(1+i) & 5 & 0 \\ -2 & -2-(1+i) & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 3-i & 5 & 0 \\ -2 & -3-i & 0 \end{array}\right]$$

mult row 1 by $3+i$ mult row 2 by 5

$$\sim \left[\begin{array}{cc|c} 10 & 5(3+i) & 0 \\ -10 & -5(3+i) & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 3+i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$r_2 = \alpha$

$$\boxed{\lambda = 1+i, \quad \vec{r} = \begin{pmatrix} -(3+i) \\ 2 \end{pmatrix}}$$

(9) 5 Determine

a) $e^{At} = R e^{\Lambda t} R^{-1}$

$$\boxed{A = \begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix}}$$

where its given that

$$\underset{R}{\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}} \underset{e^{\Lambda t}}{\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{+t} \end{pmatrix}} \underset{R^{-1}}{\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2e^{-t} & -e^{-t} \\ -e^{t} & e^{t} \end{pmatrix}$$

$$\boxed{= \begin{pmatrix} 2e^{-t} - e^{t} & -e^{-t} + e^{t} \\ 2e^{-t} - 2e^{t} & -e^{-t} + 2e^{t} \end{pmatrix}}$$

Expect that $e^{A0} = I$ it checks.

(10)

$$A^{65} = (R \Lambda R^{-1})^{65} \qquad \boxed{A = \begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix}}$$

$$= R \Lambda^{65} R^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} (-1)^{65} & 0 \\ 0 & 1^{65} \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}}_{\Lambda} \underbrace{\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}}_{R^{-1}}$$

$$= \boxed{\begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix}} = (A)$$