

Math 2318 Exam 1. Sanders Spring 2026

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Consider the matrices.

$$\begin{aligned} A &= \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} & B &= \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} & C &= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} & D &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} & E &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \end{aligned}$$

Compute the following when defined.

$$(a) \ B + 2E \quad (b) \ D + E \quad (c) \ AC \quad (d) \ CD$$

2. Write each of the following linear systems as an augmented matrix. Reduce to echelon form by Gaussian elimination. Finally determine all solutions if there is one.

$$\begin{aligned} (a) \quad & \begin{aligned} 2x_1 + 2x_2 + 2x_3 + 4x_4 &= 6 \\ x_1 + 2x_2 + x_3 + 2x_4 &= 1 \\ x_1 + 2x_2 + x_3 + 3x_4 &= 2 \end{aligned} & (b) \quad & \begin{aligned} x_1 + 2x_2 + x_3 &= 8 \\ 2x_1 + x_2 + 2x_3 &= 10 \\ x_1 + x_2 + 2x_3 &= 9 \end{aligned} \end{aligned}$$

3. Answer the following.

$$(a) \text{ Is } \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \in \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\} ? \quad (b) \text{ Is } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix} \right\} ?$$

If true, write the given vector as a linear combination of the two spanning vectors. Please show all your work.

4. Consider the two spaces which are spanned as follows.

$$(a) \quad \mathcal{V}_a = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} \quad (b) \quad \mathcal{V}_b = \text{span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \right\}$$

For each space, determine its **standard basis** and its **dimension**.

5. Consider the following two matrices A each which defines a linear operator $\mathcal{L} : \mathbb{R}^m \rightarrow \mathbb{R}^n$ via matrix multiplication; i.e. $\mathcal{L}(\mathbf{x}) = A\mathbf{x}$.

$$(a) \quad \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{pmatrix} \quad (b) \quad \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}$$

Determine a basis for $\mathcal{L}$'s **null space** and also compute the standard basis for $\mathcal{L}$'s **range space**.