①

Snider E2
2318 Sp 26

1 Find inverse (you choose method)

a) $A=\begin{pmatrix}2 & 2\\ 3 & 1\end{pmatrix}$

By Cramer's $\quad A^{-1} = \dfrac{1}{2-6}\begin{pmatrix}1 & -2\\ -3 & 2\end{pmatrix}$

or by elimination

$$\left[\begin{array}{cc|cc} 2 & 2 & 1 & 0\\ 3 & 1 & 0 & 1\end{array}\right] \sim \left[\begin{array}{cc|cc} 6 & 6 & 3 & 0\\ 6 & 2 & 0 & 2\end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 6 & 6 & 3 & 0\\ 0 & 4 & 3 & -2\end{array}\right] \sim \left[\begin{array}{cc|cc} 4 & 4 & 2 & 0\\ 0 & 4 & 3 & -2\end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 4 & 0 & -1 & 2\\ 0 & 4 & 3 & -2\end{array}\right] \Rightarrow A^{-1} = \begin{pmatrix} -\frac{1}{4} & \frac{1}{2}\\ \frac{3}{4} & -\frac{1}{2}\end{pmatrix}$$

I'd use Cramer's rule here.

b) $A=\begin{pmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 1 & 3 & 2\end{pmatrix}$ Cramer's says

$A^{-1} = \dfrac{1}{\det(A)} C^T$

(2)

Get det(A) by criss-cross

$$\det\begin{pmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 1 & 3 & 2\end{pmatrix} = [6+2+6] - [3+6+4]$$

$$= 1$$

$C_{ij} = \underline{\underline{(-1)^{i+j}}}\det(M_{ij})$

$$C = \begin{pmatrix}+0 & -2 & +3\\ -(-1) & +1 & -2\\ +(-1) & -0 & +1\end{pmatrix}$$ ugh!

$$\boxed{A^{-1} = \frac{1}{1}\begin{pmatrix}0 & 1 & -1\\ -2 & 1 & 0\\ 3 & -2 & 1\end{pmatrix}}$$

I wouldn't do this way.

By elimination

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 2 & 3 & 2 & 0 & 1 & 0\\ 1 & 3 & 2 & 0 & 0 & 1\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 0 & -2 & 1 & 0\\ 0 & 2 & 1 & -1 & 0 & 1\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 0 & -2 & 1 & 0\\ 0 & 0 & 1 & 3 & -2 & 1\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1 & 1 & 0 & -2 & 2 & -1\\ 0 & 1 & 0 & -2 & 1 & 0\\ 0 & 0 & 1 & 3 & -2 & 1\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 0 & 1 & -1\\ 0 & 1 & 0 & -2 & 1 & 0\\ 0 & 0 & 1 & 3 & -2 & 1\end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{pmatrix}0 & 1 & -1\\ -2 & 1 & 0\\ 3 & -2 & 1\end{pmatrix}}$$

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2 Calc dets

a $\det\begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 4 & 2 & 4 \end{pmatrix} = [24+4+4]-[16+6+4]$

$32-26 = \boxed{6}$

criss-cross this one

could but don't want to use cofactors.

b) $\det\begin{pmatrix} 1 & 1 & 3 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{pmatrix}$ ← cofactor along row 2

$= 2(-1)^{2+1}\det\begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix} + 0 + 0 +$

$+1(-1)^{2+4}\det\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ ← criss-cross these

$= -2\{[2+12+4]-[8+2+6]\}$ $\quad 18 - 16$

$+ \{[2+4+6]-[12+4+1]\}$ $\quad 12-17$

$-2(2)-5 = \boxed{-9}$

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3 Use <u>only</u> Cramer's rule

a) $\begin{pmatrix} \pi & 1 \\ 1 & \pi \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$x_1 = \frac{1}{\pi^2-1}\det\begin{pmatrix} 2 & 1 \\ 1 & \pi \end{pmatrix} = \boxed{\frac{2\pi-1}{\pi^2-1}}$$

$$x_2 = \frac{1}{\pi^2-1}\det\begin{pmatrix} \pi & 2 \\ 1 & 1 \end{pmatrix} = \boxed{\frac{\pi-2}{\pi^2-1}}$$

b) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\det\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix} = 2\det\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = -2$$

$$x_1 = \frac{1}{-2}\det\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \frac{2}{-2} = \boxed{-1}$$

$$x_2 = \frac{1}{-2}\det\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \frac{-0}{-2} = \boxed{0}$$

$$x_3 = \frac{1}{-2}\det\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \frac{-2}{-2} = \boxed{+1}$$

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4 Find evals & evecs

$$\det(A-\lambda I) = \det\begin{pmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{pmatrix}$$

$$= (3-\lambda)^2 - 1 = 0$$

$$\boxed{\lambda = 3 \pm 1}$$

For evecs

$\boxed{\lambda = 2}$

$$(A-\lambda I)\vec{r} = \vec{0}$$

$$\left[\begin{array}{cc|c} 3-2 & 1 & 0 \\ 1 & 3-2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$r_1 + \alpha = 0$

$r_2 = \alpha$

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} -\alpha \\ \alpha \end{pmatrix}$$

$$\boxed{r = \begin{pmatrix} -1 \\ 1 \end{pmatrix}}$$

Take $\alpha = 1$ WLOG

$\boxed{\lambda = 4}$

$$\left[\begin{array}{cc|c} 3-4 & 1 & 0 \\ 1 & 3-4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\boxed{r = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

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b $\det(A-\lambda I) = \det\begin{pmatrix} 1-\lambda & 2 \\ -1 & 3-\lambda \end{pmatrix}$

$= (\lambda-3)(\lambda-1)+2 = \lambda^2-4\lambda+5 = 0$

$\lambda = \dfrac{4 \pm \sqrt{16-4\cdot5}}{2} = \dfrac{4\pm 2i}{2}$

$\boxed{\lambda = 2 \pm i}$

$\lambda = 2+i$ $\quad \left[\begin{array}{cc|c} 1-(2+i) & 2 & 0 \\ -1 & 3-(2+i) & 0 \end{array}\right]$

$\sim \left[\begin{array}{cc|c} -(1+i) & 2 & 0 \\ -1 & 1-i & 0 \end{array}\right] \times(1-i)$

$\sim \left[\begin{array}{cc|c} -2 & 2(1-i) & 0 \\ -1 & 1-i & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & (1-i) & 0 \\ 0 & 0 & 0 \end{array}\right]$

$r_2 = \alpha$

$\boxed{r = \begin{pmatrix} 1-i \\ 1 \end{pmatrix}}$

⑦

$\lambda = 2-i$ $\left[\begin{array}{cc|c} 1-(2-i) & 2 & 0 \\ -1 & 3-(2-i) & 0 \end{array}\right]$

$-(1-i)$

$\sim \left[\begin{array}{cc|c} -1+i & 2 & 0 \\ -1 & 1+i & 0 \end{array}\right] \times (1+i)$

$\sim \left[\begin{array}{cc|c} -2 & 2(1+i) & 0 \\ -1 & 1+i & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & (1+i) & 0 \\ 0 & 0 & 0 \end{array}\right]$

$r_2 = \alpha$

$\boxed{r = \begin{pmatrix} 1+i \\ 1 \end{pmatrix}}$

5 $R^{-1}AR = \underset{R^{-1}}{\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}} \underset{A}{\begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix}} \underset{R}{\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}} = \underset{\Lambda}{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$

$A = R\Lambda R^{-1}$

$e^{At} = Re^{\Lambda t}R^{-1}$

$= \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{t} & 0 \\ 0 & e^{-t} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = (\text{over})$

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{t} & -e^{t} \\ -e^{-t} & 2e^{-t} \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 2e^{t} - e^{-t} & -2e^{t} + 2e^{-t} \\ e^{t} - e^{-t} & -e^{t} + 2e^{-t} \end{pmatrix}}$$

b) Calc A^{22}

Since $A = R \Lambda R^{-1}$

$$A^{22} = R \Lambda^{22} R^{-1}$$

$$= R \begin{pmatrix} 1^{22} & 0 \\ 0 & (-1)^{22} \end{pmatrix} R^{-1}$$

$$= R \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} R^{-1} = \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$