Math 2318 Exam 2. Sanders Spring 2026

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Find the inverse if it exists. (You may use elimination or Cramer's rule. State which.)

(a) $\begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 2 \end{pmatrix}$

2. Calculate the determinant. (If you use cofactors, state which row or column.)

(a) $\begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 4 & 2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 3 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{pmatrix}$

3. Use Cramer's rule to solve the following.

(a) $\begin{pmatrix} \pi & 1 \\ 1 & \pi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(You will not receive credit if you guess or use elimination.)

4. Find all eigenvalues and corresponding eigenvectors.

(a) $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

FYI: Part (b) has complex eigenvalues.

5. You may freely use the given fact that

$$A = \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix} \quad \Rightarrow \quad R^{-1}AR \equiv \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \Lambda.$$

Don't compute eigenvalues or eigenvectors for the matrix $A$. They are given to you above.

(a) Calculate $e^{At}$. (b) Calculate $A^{22}$.