This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Find the general solutions to the following second order differential equations by solving for each two appropriate first order, linear equations.
   \[ (a) \frac{d^2 u}{dx^2} + \frac{1}{x} \frac{du}{dx} = 0 \quad (b) \frac{d}{dx} \left( \frac{du}{dx} + \frac{1}{x} u \right) = 0 \]

Recall \( e^{\log x} = x \).

2. Determine the general solution to each of the following second order, constant coefficient differential equations.
   \[ (a) \frac{d^2 u}{dx^2} + 2 \frac{du}{dx} + 2u = 0 \quad (b) \frac{d^2 u}{dx^2} + 2 \frac{du}{dx} - 3u = 0 \]

3. Solve the following initial value problems (IVPs).
   \[ (a) \frac{d^2 u}{dx^2} - 2 \frac{du}{dx} + u = 0 \quad (b) \frac{d^2 u}{dx^2} + 4u = 0 \]
   \[ u(1) = 1, \ u'(1) = 2 \quad u(3) = 1, \ u'(3) = 2 \]

4. Find a particular solution to each of the following differential equations by using any method you wish.
   \[ (a) \frac{d^2 u}{dx^2} + u = x^2 + 1 \quad (c) \frac{d^2 u}{dx^2} + u = \sin(x) \]
   \[ (b) \frac{d^2 u}{dx^2} + u = e^x \quad (d) \frac{d^2 u}{dx^2} + u = x^2 + 1 + 2\sin(x) + 3e^x \]

5. Use Duhamel's principle to solve the following IVPs.
   \[ (a) \frac{d^2 u}{dx^2} + u = 1 \quad (b) \frac{d^2 u}{dx^2} - u = 1 \]
   \[ u(0) = 0, \ u'(0) = 0 \quad u(1) = 0, \ u'(1) = 0 \]

Note: The initial conditions are specified at \( x = 0 \) in part (a) but at \( x = 1 \) in part (b),