Math 3331 Exam 3. Sanders Fall 2017

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Consider the following finite difference schemes for finding approximate solutions to first order IVP’s of the form $\frac{dx}{dt} = f(x,t)$, $x(0) = x_0$.

(FE) $x_{n+1} = x_n + \Delta tf(x_n, t_n)$

(TR) $x_{n+1} = x_n + \frac{1}{2}\Delta t (f(x_n, t_n) + f(x_{n+1}, t_{n+1}))$

(a) Determine the local truncation error for the FE method.
(b) Determine the local truncation error for the TR method.

2. Write each scalar initial value problem as a first order system.

(a) $\frac{d^2u}{dt^2} - 3\frac{du}{dt} + 2u = 0$, $u(0) = 1$, $u'(0) = 2$.

(b) $\frac{d^3u}{dt^3} - 2\frac{d^2u}{dt^2} - \frac{du}{dt} + 2u = 0$, $u(0) = 1$, $u'(0) = 2$, $u''(0) = 3$.

3. Determine the eigenvalues for each of the following matrices.

(a) $\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

4. Both of the following matrices have eigenvalues $\lambda = 1$ and $\lambda = 2$ (You don’t need to compute these.) For each however, determine the eigenvector associated to the given eigenvalue.

(a) $\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}$

5. Solve the following initial value problem. (You may reuse your eigenvalues and eigenvectors from above.)

$$\frac{dx}{dt} = -x + 3y$$

$$\frac{dy}{dt} = -2x + 4y$$

$x(0) = 1$, $y(0) = 2$. 