Math 3331 Exam 1. Sanders Fall 2018

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Determine which if any of the given differential operators are linear. You must show your work.
   (a) $\mathcal{L}(u) = u \frac{du}{dx} + u$.
   (b) $\mathcal{L}(u) = x \frac{du}{dx} + u$.

2. Find explicit form general solutions to the following.
   (a) $\frac{du}{dx} - x^2 u^2 = 0$.
   (b) $2xu \frac{du}{dx} + 2x + u^2 = 0$.

3. Find explicit form general solutions to the following.
   (a) $xu \frac{du}{dx} - (u^2 + x^2) = 0$.
   (b) $\frac{du}{dx} + u - e^{-x} = 0$.

4. Find explicit solutions to the following initial value problems (IVPs).
   (a) $\frac{du}{dx} = u^2, \quad u(0) = 1$.
   (b) $\frac{du}{dx} + u = x, \quad u(0) = 1$.

5. Newton’s law of cooling states that a body’s temperature as a function of time, $T(t)$, satisfies
   \[
   \frac{dT}{dt} = k(T_A - T),
   \]
   where $T_A$ is the environment’s ambient temperature, and $k$ is a rate constant.

Suppose we know the following.

$T_A = 50, \quad T(0) = 70, \quad T(1) = 60$.

(a) Determine the rate constant $k$.

(b) Now solve the IVP for $T(t)$. 