Math 3331 Exam 3. Sanders Fall 2018

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Determine (1) the natural matrix norm, and (2) the Frobenius norm for the following.
   (a) \[
   \begin{pmatrix}
   1 & 1 \\
   0 & 1
   \end{pmatrix}
   \].
   (b) \[
   \begin{pmatrix}
   1 & 2 \\
   1 & 1
   \end{pmatrix}
   \].

   Recall that \( ||A||^2 \) is given by the largest eigenvalue of \( A^T A \).

2. Determine (1) the eigenvalues, and (2) the corresponding eigenvectors for the following.
   (a) \[
   \begin{pmatrix}
   -3 & 2 \\
   -4 & 3
   \end{pmatrix}
   \].
   (b) \[
   \begin{pmatrix}
   0 & 1 \\
   -1 & 2
   \end{pmatrix}
   \].

3. Determine \( e^{At} \) when \( A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \). You may use the facts that \( \lambda_1 = 1 \), \( r_1 = (1, 1)^T \) and \( \lambda_2 = 2 \), \( r_2 = (1, 2)^T \).

4. Use the matrices in this similarity transformation

   \[
   S^{-1}AS \equiv \frac{1}{2} \begin{pmatrix}
   1 & -1 \\
   1 & 1
   \end{pmatrix} \begin{pmatrix}
   0 & 1 \\
   1 & 0
   \end{pmatrix} \begin{pmatrix}
   1 & 1 \\
   -1 & 1
   \end{pmatrix} = \begin{pmatrix}
   -1 & 0 \\
   0 & 1
   \end{pmatrix} \equiv \Lambda
   \]

   to solve the initial value problem

   \[
   \frac{dx}{dt} = y, \quad x(0) = 1,
   \]

   \[
   \frac{dy}{dt} = x, \quad y(0) = 0.
   \]

5. The matrix \( A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \) is not diagonalizable.

   (a) Find a matrix \( S \) such that \( S^{-1}AS = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \equiv J \).

   (b) Determine \( e^{At} \).