Math 3331 Exam 1. Sanders Fall 2022

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. We’ve considered four basic classes of first order ODE’s we can in principle solve. They are: (1) Separable, (2) Linear, (3) Homogeneous, (4) Exact. Classify each of the following but do not solve. An equation may be in more than one class (and you must list all classes if so), or it may be in none. No guessing – show your work!

   (a) \( \frac{du}{dx} + \sin(x)u^2 = 0 \)  
   (b) \( x^2 \frac{du}{dx} + xu + x^2 = 0 \)  
   (c) \( \frac{du}{dx} + u + x^2 = 0 \)  
   (d) \( (u^2 + x + 1) \frac{du}{dx} + u + 2x = 0 \)

2. Find the **explicit** form general solution to the following first order ODEs.

   (a) \( \frac{du}{dx} - 2xu^2 = 0 \)  
   (b) \( (2u + x) \frac{du}{dx} + u = 0 \)

   Hint for (b). The quadratic formula may prove useful to write the solution in explicit form.

3. Find the **explicit** form general solution to these as well.

   (a) \( \frac{du}{dx} - 2xu = xe^{x^2} \)  
   (b) \( \frac{du}{dx} = \frac{u^2 + xu}{xu} \)

4. Recall the cylindrical water tank with a small drain hole given on your applications homework. There we derived the following IVP from Bernoulli’s principle.

   \[
   \frac{dz}{dt} = -k\sqrt{z}, \quad z(0) = z_0 > 0,
   \]

   where \( z(t) \) is the depth of the water at time \( t \) and \( k \) is a constant. At \( t = 0 \) we measure \( z_0 \), and at \( t = 1 \) we again measure the water depth getting \( z(1) = z_1 \).

   (a) Determine the constant \( k \) in the IVP in terms of \( z_0 \) and \( z_1 \). Answer: \( k = 2(\sqrt{z_0} - \sqrt{z_1}) \).

   (b) Determine the time \( t_\ast \) it takes for the tank to empty in terms of \( z_0 \) and \( z_1 \).

5. Find the general solution of the following second order differential equations by using the given factorization.

   (a) \( \left( \frac{d}{dx} + I \right) \left( \frac{du}{dx} + u \right) = \frac{d^2 u}{dx^2} + 2 \frac{du}{dx} + u = 0 \).

   (b) \( \left( \frac{d}{dx} - I \right) \left( \frac{du}{dx} - 3u \right) = \frac{d^2 u}{dx^2} - 4 \frac{du}{dx} + 3u = e^x \).