1. Consider the following finite difference schemes for finding approximate solutions to first order IVP’s of the form \(\frac{dx}{dt} = f(x, t)\), \(x(0) = x_0\).

(BE) \[x_{n+1} = x_n + \Delta t f(x_{n+1}, t_{n+1})\]

(TR) \[x_{n+1} = x_n + \frac{1}{2} \Delta t (f(x_n, t_n) + f(x_{n+1}, t_{n+1}))\]

(a) Determine the local truncation error for the BE method.

(b) Determine the local truncation error for the TR method.

2. Write each scalar initial value problem as a first order system.

(a) \[\frac{d^2 u}{dt^2} + 2 \frac{du}{dt} + u = 0, \quad u(0) = 1, \quad u'(0) = 0\]

(b) \[\frac{d^3 u}{dt^3} + \frac{d^2 u}{dt^2} + \frac{du}{dt} + u = 0, \quad u(0) = 3, \quad u'(0) = 2, \quad u''(0) = 1\]

3. Determine all eigenvalues and eigenvectors for the following matrices. Also, explain why or why not the given matrix is diagonalizable.

(a) \[
\begin{pmatrix}
3 & -1 \\
2 & 0
\end{pmatrix}
\]

(b) \[
\begin{pmatrix}
0 & 1 \\
-1 & -2
\end{pmatrix}
\]

4. Both of the following matrices \(A\) have eigenvalues \(\lambda = 1\) and \(\lambda = 2\) (you may use these freely). Compute the closed form for \(e^{At}\). Hint: Recall \(e^{At} = Re^{At}R^{-1}\).

(a) \(A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}\)

(b) \(A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}\)

5. Use the fact that the matrix

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

has \(\lambda = -1\), \(r = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\) and \(\lambda = 1\), \(r = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\),

to solve the initial value problem

\[
\begin{align*}
\frac{dx}{dt} &= y \\
\frac{dy}{dt} &= x \\
x(0) &= 1, \quad y(0) = 0
\end{align*}
\]