1. For each of the following differential operators, \( L(u) \), state its order, and then determine (show your work) whether or not it is linear in the dependent variable \( u \).

(a) \( L(u) = u \frac{du}{dx} + u \)

(b) \( L(u) = x \frac{du}{dx} + u \)

(c) \( L(u) = \frac{d^2u}{dx^2} + e^x \frac{du}{dx} \)

(d) \( L(u) = \frac{d^2u}{dx^2} + \left( \frac{du}{dx} \right)^2 \)

Recall linear implies both \( L(u_1 + u_2) = L(u_1) + L(u_2) \) and \( L(cu) = cL(u) \). Don’t forget to state \( L \)'s order!

2. Find the explicit general solution to the following first order ODEs.

(a) \( \frac{du}{dx} = u^2 - u \)

(b) \( x^2 \frac{du}{dx} + u^2 - xu = 0 \)

3. Find the explicit general solution to these.

(a) \( \frac{du}{dx} + \frac{1}{x}u = e^x \)

(b) \( xe^u \frac{du}{dx} + x + e^u = 0 \)

Recall Newton’s law of cooling

\[
\frac{dT}{dt} = k_b(T_A - T),
\]

where \( T(t) \) is the temperature of a given body at time \( t \), \( T_A \) is the surrounding ambient temperature, and the rate constant \( k_b \) is characteristic of the given body.

4. A man has been killed. The police forensic team arrives at the crime scene at 10 am. They immediately measure the scene’s ambient temperature, \( T_A = 70^\circ F \), and the dead man’s body core temperature, \( T(0) = 80^\circ F \). One hour later they again measure the man’s core temperature and got \( T(1) = 75^\circ F \). Assume the man’s core temperature was \( 98^\circ F \) at the moment he was killed. Use Newton’s law of cooling to determine when the crime was committed? (Leave your answer in log hours relative to 10 am.)

5. Determine the general solution to each of the following homogeneous, second order, constant coefficient differential equations. (You’re not asked to factor these.)

(a) \( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = 0 \)

(b) \( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0 \)

(c) \( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - y = 0 \)

(d) \( \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 5y = 0 \)