Math 3331 Exam 1. Sanders Spring 2022

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Determine which of the following differential operators $D(u)$ are linear and which are not. You must show your work. Also, state $D$’s order.

   (a) $D(u) \equiv \frac{du}{dx} + x u$
   (b) $D(u) \equiv \frac{du}{dx} + e^u$
   (c) $D(u) \equiv \frac{d^2u}{dx^2} + e^x \frac{du}{dx}$
   (d) $D(u) \equiv v \frac{d^2u}{dx^2}$

2. Find the explicit form general solution to the following first order ODEs.

   (a) $\frac{du}{dx} - u^2 = 0$
   (b) $2xu \frac{du}{dx} + 2x + u^2 = 0$

3. Find the explicit form general solution to these as well.

   (a) $\frac{du}{dx} - 2x u = e^{x^2}$
   (b) $\frac{du}{dx} = \frac{u^2 + xu}{xu}$

Recall Newton’s law of cooling

$$\frac{dT}{dt} = k_b (T_A - T),$$

where $T(t)$ is the temperature of a given body at time $t$, $T_A$ is the surrounding ambient temperature, and the rate constant $k_b$ is characteristic of the given body.

4. A man has been killed. The police forensic team arrives at the crime scene at 12 noon. They immediately measure the scene’s ambient temperature, $T_A = 70\,^\circ F$, and the dead man’s body core temperature, $T(0) = 80\,^\circ F$. One hour later they again measure the man’s core temperature and got $T(1) = 75\,^\circ F$. Use Newton’s law of cooling to do the following.

   (a) Use $T_A$, $T(0)$ and $T(1)$ to determine the rate constant $k_b$.
   (b) How many hours before noon was the man’s core temperature $98\,^\circ F$?

(Leave your answer in terms of logs.)

5. Find the general solution of the following second order differential equations by using the given factorization.

   (a) $\frac{d^2u}{dx^2} + 3 \frac{du}{dx} + 2u = \left( \frac{d}{dx} + I \right) \left( \frac{du}{dx} + 2u \right) = 1.$
   (b) $\frac{d^2u}{dx^2} - 2 \frac{du}{dx} + u = \left( \frac{d}{dx} - I \right) \left( \frac{du}{dx} - u \right) = 0.$