1. Factorize the following second order differential operators into the composition of two first order operators.

   (a) \( \frac{d^2 u}{dx^2} + u \). 
   (b) \( \frac{d^2 u}{dx^2} - \frac{du}{dx} - 2u \).

2. Determine the general solution (homogeneous solution + particular solution) to each of the following by using the method of guessing.

   (a) \( \frac{d^2 u}{dx^2} = e^{2x} \). 
   (b) \( \frac{d^2 u}{dx^2} = x + 1 \). 
   (c) \( \frac{d^2 u}{dx^2} - \frac{du}{dx} = 1 \). 
   (d) \( \frac{d^2 u}{dx^2} - \frac{du}{dx} = e^x \).

3. Use Duhamel to find the solution of each of the following initial value problems.

   (a) \( \frac{d^2 u}{dx^2} + u = x, \quad u(0) = u_x(0) = 0 \). 
   (b) \( \frac{d^2 u}{dx^2} - u = e^x, \quad u(0) = u_x(0) = 0 \).

4. Write each of the following scalar differential equations as a first order system.

   (a) \( \frac{d^2 u}{dt^2} - 2 \frac{du}{dt} - u = 0 \). 
   (b) \( \frac{d^2 u}{dt^2} + u \frac{du}{dt} = 0 \). 
   (c) \( \frac{d^2 u}{dt^2} = \left( \frac{du}{dt} \right)^2 + e^u \). 
   (d) \( \frac{d^3 u}{dt^3} = \frac{d^2 u}{dt^2} + u \).

5. Consider the matrix \( A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \).

   (a) Determine the eigenvalues of \( A \).
   (b) Determine the eigenvectors of \( A \).
   (c) Determine the matrix \( e^{At} \).
   (d) Use part (c) to solve the following initial value problem

\[
\begin{cases}
\frac{du}{dt} = 2u + v, & u(0) = 0, \\
\frac{dv}{dt} = u + 2v, & v(0) = 1.
\end{cases}
\]