1. Factorize the following second order differential operators into the composition of two first order operators.
   (a) $\frac{d^2 u}{dx^2} + \frac{du}{dx} - 6u$.  
   (b) $\frac{d^2 u}{dx^2} - 2\frac{du}{dx} + 2u$.

2. Determine the general solution (homogeneous solution + particular solution) to each of the following by using the method of guessing.
   (a) $\frac{d^2 u}{dx^2} + \frac{du}{dx} = e^x$.  
   (b) $\frac{d^2 u}{dx^2} + \frac{du}{dx} = x + 1$.  
   (c) $\frac{d^2 u}{dx^2} + \frac{du}{dx} = e^{-x}$.  
   (d) $\frac{d^2 u}{dx^2} + \frac{du}{dx} = \sin(x)$.

   (Since the LHSs are all the same, you only need to find the homogeneous solution once.)

3. Use Duhamel to find the solution of each of the following initial value problems.
   (a) $\frac{d^2 u}{dx^2} - u = 1$,  $u(0) = u_x(0) = 0$.  
   (b) $\frac{d^2 u}{dx^2} - u = xe^x$,  $u(0) = u_x(0) = 0$.

4. Write each of the following scalar differential equations as a first order system.
   (a) $\frac{d^2 u}{dt^2} - 2\frac{du}{dt} - 3u = 0$.  
   (b) $\frac{d^2 u}{dt^2} - u^2 \frac{du}{dt} = 0$.  
   (c) $\frac{d^2 u}{dt^2} = \left(\frac{du}{dt}\right)^3$.  
   (d) $\frac{d^3 u}{dt^3} = \frac{du}{dt} + 3u$.

5. Consider the matrix $A = \begin{pmatrix} -2 & 1 \\ -12 & 5 \end{pmatrix}$.
   (a) Determine the eigenvalues of $A$.
   (b) Determine the eigenvectors of $A$.
   (c) Determine the matrix $e^{At}$.
   (d) Use part (c) to solve the following initial value problem:

   \[
   \begin{cases}
   \frac{du}{dt} = -2u + v, & u(0) = 1, \\
   \frac{dv}{dt} = -12u + 5v, & v(0) = 0.
   \end{cases}
   \]