

Math 3331 Exam 2. Sanders Spring 2025

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Solve the following initial value problems (IVPs) by **any method**.

(a) $\frac{d^2u}{dx^2} - 2\frac{du}{dx} + u = 0, \quad u(0) = 1, \quad u_x(0) = 2.$

(b) $\frac{d^2u}{dx^2} + u = 0, \quad u(1) = 1, \quad u_x(1) = 2.$ Note, this condition is specified at $x = 1$.

2. Determine the general solution (homogeneous solution + particular solution) to each of the following by using the **method of guessing**.

(a) $\frac{d^2u}{dx^2} + \frac{du}{dx} = 1.$ (c) $\frac{d^2u}{dx^2} + \frac{du}{dx} = e^{-x}.$

(b) $\frac{d^2u}{dx^2} + \frac{du}{dx} = e^x.$ (d) $\frac{d^2u}{dx^2} + \frac{du}{dx} = \cos(x).$

(Since the LHs are all the same, you only need to find the homogeneous solution once.)

3. Use **Duhamel's method** to find the solution of each of the following IVPs.

(a) $\frac{d^2u}{dx^2} - u = 1, \quad u(0) = u_x(0) = 0.$ (b) $\frac{d^2u}{dx^2} = x^2, \quad u(1) = u_x(1) = 0.$

Notice in part (b) the initial condition is specified at $x = 1$.

4. Write each of the following scalar differential equations as a first order system.

(a) $\frac{d^2u}{dt^2} + \frac{du}{dt} - 2u = 0.$ (c) $\frac{d^2u}{dt^2} = \left(\frac{du}{dt}\right)^2 e^u.$

(b) $\frac{d^2u}{dt^2} - u \frac{du}{dt} = 0.$ (d) $\frac{d^3u}{dt^3} = \frac{du}{dt} + 3u.$

5. Consider the matrix $A = \begin{pmatrix} -3 & 2 \\ -4 & 3 \end{pmatrix}.$

(a) Determine the eigenvalues of A . (b) Determine the eigenvectors of A .

(c) Determine the matrix e^{At} .

(d) Use part (c) to solve the following initial value problem:

$$\begin{cases} \frac{du}{dt} = -3u + 2v, & u(0) = 0, \\ \frac{dv}{dt} = -4u + 3v, & v(0) = 1. \end{cases}$$