

Math 3331 Exam 2. Sanders Spring 2026

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Solve the following initial value problems (IVPs) by any method.

(a) $\frac{d^2u}{dx^2} + 2\frac{du}{dx} + u = 0, \quad u(0) = 1, \quad u_x(0) = 2.$

(b) $\frac{d^2u}{dx^2} + u = 0, \quad u(1) = 1, \quad u_x(1) = 0.$ Notice this IVP is specified at $x = 1$.

2. Use the method of guessing to find the general solution (homogeneous solution + particular solution) to each of the following.

(a) $\frac{d^2u}{dx^2} - \frac{du}{dx} = e^{-x}.$ (c) $\frac{d^2u}{dx^2} - \frac{du}{dx} = e^x.$

(b) $\frac{d^2u}{dx^2} - \frac{du}{dx} = x + 1.$ (d) $\frac{d^2u}{dx^2} - \frac{du}{dx} = \cos(x).$

(You may freely use the fact that homogeneous solution to each is $u_h(x) = c_1 + c_2e^x$.)

3. Use Duhamel to find the solution for each of the following IVPs.

(a) $\frac{d^2u}{dx^2} = x, \quad u(0) = u_x(0) = 0.$ (b) $\frac{d^2u}{dx^2} - 2\frac{du}{dx} + u = e^x, \quad u(0) = u_x(0) = 0.$

4. Write each of the following scalar differential equations as a first order system.

(a) $\frac{d^2u}{dt^2} + 2\frac{du}{dt} + 3u = 0.$ (c) $\frac{d^2u}{dt^2} = \left(\frac{du}{dt}\right)^4 + \sin(u).$

(b) $\frac{d^2u}{dt^2} - u\frac{du}{dt} + u^2 = 0.$ (d) $\frac{d^3u}{dt^3} = \frac{d^2u}{dt^2} + u.$

5. Consider the matrix $A = \begin{pmatrix} -1 & 3 \\ -2 & 4 \end{pmatrix}.$

(a) Determine the eigenvalues of A .

(b) Determine the associated eigenvectors of A .

(c) Determine the matrix e^{At} .

(d) Use part (c) to solve the following initial value problem:

$$\begin{cases} \frac{du}{dt} = -u + 3v, & u(0) = 1, \\ \frac{dv}{dt} = -2u + 4v, & v(0) = 0. \end{cases}$$