

Math 3331 Final Exam. Sanders Spring 2026

This exam has seven problems and all problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Find the explicit solution for the following initial value problems.

$$(a) \frac{du}{dx} = xe^{-u}, \quad u(0) = 1. \quad (b) \frac{du}{dx} + 2u = e^x, \quad u(0) = 2.$$

2. Find the general solution to the following. You may leave your answer in implicit form.

$$(a) (u^2 + x) \frac{du}{dx} + (u + 1) = 0. \quad (b) \frac{du}{dx} = \frac{u^2 + x^2}{ux}.$$

3. Consider two storage tanks, both initially holding ten gallons of fresh water. There is a pipe of briny water flowing into the first tank at the rate of one gallon per minute. The salt concentration in this first pipe is fixed at one pound of salt per gallon of water. There is second pipe flowing from the first tank into the second tank and a third pipe flowing out of the second tank, both also with flow rates of one gallon per minute. Let $s_1(t)$ and $s_2(t)$ denote the amount of salt (in pounds) at time t in the first and second tank respectively.

(a) Determine the two first order IVPs for $s_1(t)$ and $s_2(t)$. (10 pts)

(b) Solve for $s_1(t)$. (5 pts)

(c) Solve for $s_2(t)$. (5 pts)

4. Find the general solution to the following second order ODEs. You may use either the method guessing or the method of factoring.

$$(a) \frac{d^2u}{dx^2} - \frac{du}{dx} = 1. \quad (b) \frac{d^2u}{dx^2} - 2\frac{du}{dx} + u = e^x.$$

5. Use Duhamel to solve the following second order inhomogeneous IVPs.

$$(a) \frac{d^2u}{dx^2} - u = 1. \quad u(0) = u_x(0) = 0. \quad (b) \frac{d^2u}{dx^2} = xe^x. \quad u(0) = u_x(0) = 0.$$

No method of guessing here. Only Duhamel's method will be graded.

6. The matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 2 & 2 & 4 \end{pmatrix}$ has eigenvalues, $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 3$.

(a) (10 pts) Determine R so that $R^{-1}AR = \Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$.

(b) (5 pts) Compute e^{At} .

(c) (5 pts) Use part (b) to solve the following IVP.

$$\begin{cases} \frac{dx}{dt} = x + z, & x(0) = 1, \\ \frac{dy}{dt} = y - 2z, & y(0) = 0, \\ \frac{dz}{dt} = 2x + 2y + 4z & z(0) = 0. \end{cases}$$

7. The matrix $A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$ is not diagonalizable.

(a) (10 pts) Determine S such that $S^{-1}AS = J$ where $J = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$.

(b) (5 pts) Compute e^{At} .

(c) (5 pts) Use part (b) to solve the following IVP.

$$\begin{cases} \frac{dx}{dt} = 2x + y, & x(0) = 1, \\ \frac{dy}{dt} = -x + 4y, & y(0) = 0. \end{cases}$$