Math 3335 Exam 1. Sanders Spring 2018

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Suppose vector addition and scalar multiplication for two component column matrices are defined as usual:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \ \alpha \in \mathbb{R} \quad \Rightarrow \quad \mathbf{x} + \mathbf{y} \equiv \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}, \quad \alpha\mathbf{x} \equiv \begin{pmatrix} \alpha x_1 \\ \alpha x_2 \end{pmatrix}.$$

With this, determine whether or not the following sets define vector spaces.

(a) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : \ x_2 = 0 \right\}$ (b) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : \ 2x_1 + 3x_2 = 0 \right\}$

(c) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : \ x_1 + x_2 = 1 \right\}$ (d) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : \ x_2 \geq 0 \right\}$

You only need check if these sets are closed under scalar multiplication and vector addition.

2. Both formulae below define norms on $\mathbb{R}^2$. Show they satisfy the triangle inequality.

(a) $||\mathbf{x}|| \equiv \max(|x_1|, |x_2|)$. (b) $||\mathbf{x}|| \equiv 2|x_1| + 3|x_2|$.

You are free to use the triangle inequlity for real numbers, $a, \ b \in \mathbb{R}, \ |a + b| \leq |a| + |b|$.

3. Let $\mathbf{b}_1 = (1, 2, 3)^t$, $\mathbf{b}_2 = (3, 2, 1)^t$ and $\mathbf{x} = (1, 1, 1)^t$.

(a) Project $\mathbf{x}$ into span $\{ \mathbf{b}_1 \}$. (a) Project $\mathbf{x}$ into span $\{ \mathbf{b}_1, \mathbf{b}_2 \}$.

4. Recall the projection, $\mathbf{P}_{\mathcal{S}}(\mathbf{x})$, satisfies two defining properties:

(1) $\mathbf{P}_{\mathcal{S}}(\mathbf{x}) \in \mathcal{S}$. (2) $(\mathbf{x} - \mathbf{P}_{\mathcal{S}}(\mathbf{x})) \cdot \mathbf{z} = 0$ for every $\mathbf{z} \in \mathcal{S}$.

(a) Prove $\mathbf{P}_{\mathcal{S}}(\mathbf{x}) = \mathbf{x}$ if and only if $\mathbf{x} \in \mathcal{S}$. (b) Prove the projection is unique.

5. Consider three vectors from $\mathbb{R}^3$: $\mathbf{x} = (1, 1, 1)^t$, $\mathbf{y} = (1, 2, 1)^t$ and $\mathbf{z} = (0, 1, 2)^t$. Compute the following.

(a) $\mathbf{y} \cdot \mathbf{z}$ (b) $\mathbf{x} \times \mathbf{z}$ (c) $\mathbf{y} \cdot (\mathbf{x} \times \mathbf{z})$ (d) $\mathbf{y} \cdot (2\mathbf{x} \times 3\mathbf{z})$