1. Prove the following.
   (a) If event $A \subseteq B$, then $P(A) = P(A|B)P(B)$.
   (b) If events $A$ and $B$ are independent, then $P(A|B) = P(A)$.
   (c) If events $A$ and $B$ are independent, then $A$ and $B^c$ are also independent.
   (d) If $X$ and $Y$ are independent random variables, then their joint distribution $F_{X,Y}(x,y)$ is equal to the product of their marginal distributions $F_X(x)$ and $F_Y(y)$.

2. Use the law of total probability to solve the following.
   (a) Historically, the stock market goes up 60% of the time following a democrat election victory. It goes up 80% of the time following a republican victory. 60% of elections are won by the democrats. What percentage of the time does the stock market go up following an election?
   (b) The Cougar basketball team’s shooting percentage is 50% when they play on the road. They shoot 75% when they play at home. Given that they shot 60% for the season, what percentage of their games are home games?

3. The function
   $$f_{X,Y}(x,y) = \begin{cases} 
   x + y & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\
   0 & \text{otherwise}
   \end{cases}$$

   is the joint density for random variables $X$ and $Y$. (a) Compute $\mu_X$; (5 points). (b) Compute $\mu_Y$; (5 points). (c) Compute $\text{Cov}(X,Y)$; (10 points).

4. Let $f_{X,Y}(x,y)$ be the joint density given in problem 3 above. (a) Compute the marginal density $f_X(x)$. (b) Compute the marginal distribution $F_X(x)$.

5a. Suppose $h(x) = \sqrt{|x|}$. Show that $\{x \in \mathbb{R} : h(x) \leq y\} = \begin{cases} 
   \emptyset & \text{if } y < 0 \\
   [-y^2, y^2] & \text{if } y \geq 0.
   \end{cases}$

   b. Given that $X \sim U(-1, 1)$ and $Y = \sqrt{|X|}$ determine the distribution function for $Y$.

   c. Suppose $h(x) = |x - 1|$. Show that $\{x \in \mathbb{R} : h(x) \leq y\} = \begin{cases} 
   \emptyset & \text{if } y < 0 \\
   [1 - y, y + 1] & \text{if } y \geq 0.
   \end{cases}$

   d. Given that $X \sim Exp(1)$ and $Y = |X - 1|$ determine the distribution function for $Y$. 