Math 3364 Exam 3. Sanders Fall 2017

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, last name first, and student id number on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. Each function \( f(z) \) below has an isolated singularity at \( z = 0 \). Determine its full Laurent expansion there, and classify the singularity; (is it removable, a pole, or essential).
   \( f(z) = \frac{1}{z^2(1-z)} \) \hspace{1cm} \( f(z) = e^z + e^{1/z} \)

2. Each function \( f(z) \) below has a pole at \( z = 0 \). Determine the pole’s order there and calculate its associated residue.
   \( f(z) = \frac{1}{ze^z-1} \) \hspace{1cm} \( f(z) = \frac{z}{(e^z-1)^2} \)
   I’d prefer you do these by long division.

3. The function \( f(z) \equiv \frac{1}{z \sin z} \) has a pole at \( z = 0 \) as well as poles at \( z = n\pi \), with \( n = \pm 1, \pm 2, \ldots \).
   \( f(z) \)’s residue at \( z = 0 \).
   \( f(z) \)’s residue at \( z = n\pi, n = \pm 1, \pm 2, \ldots \).

4. Use residues to calculate the value of the following.
   \( \int_{0}^{2\pi} \cos \theta \, d\theta \) \hspace{1cm} \( \int_{0}^{2\pi} \frac{1}{2 + \cos \theta} \, d\theta \)
   Obviously the answer for (a) is zero. Conclude residues also gives zero.

5. Use residues to calculate the value of the following.
   \( \int_{-\infty}^{\infty} \frac{1}{x^2 + 1} \, dx \) \hspace{1cm} \( \int_{-\infty}^{\infty} \frac{1}{x^2 + 4} \, dx \)
   Do not explicitly integrate. Use residues.