①

1 Find solns

a) $z^4 = 1 \qquad (1) = e^{0 + 2n\pi i}$

$$z = e^{\frac{0 + n\pi i}{2}}, \quad n = 0, 1, 2, 3$$

$$z_0 = e^0 = 1$$

$$z_1 = e^{\frac{\pi i}{2}} = (i)$$

$$z_2 = e^{\pi i} = -1$$

$$z_3 = e^{\frac{3\pi i}{2}} = -i$$

b) $z^2 = 4i = 2^2 e^{\frac{\pi i}{2} + 2n\pi i}$

$$z = 2e^{\frac{\pi i}{4} + n\pi i}, \quad n = 0, 1$$

$$z_0 = 2e^{\frac{\pi i}{4}} = \left(2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)$$

$$z_1 = 2e^{\frac{5\pi i}{4}} = \left(-2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)$$

(2)

2 Find $Re(f)$ & $Im(f)$

a) $f(z) = (z-1)^2 = (x-1+iy)^2$

$$= \boxed{(x-1)^2 - y^2 + 2(x-1)y\,i}$$

$$u \quad + \quad iv$$

b) $f(z) = \dfrac{z}{z+1} \cdot \dfrac{\overline{(z+1)}}{\overline{(z+1)}}$

$$= \frac{|z|^2 + z}{|z+1|^2} = \frac{x^2+y^2+x+iy}{(x+1)^2+y^2}$$

$$= \boxed{\frac{x^2+y^2+x}{(x+1)^2+y^2} + i\,\frac{y}{(x+1)^2+y^2}}$$

$$u \quad + \quad iv$$

(3)

3. C-R to check if diff'b'l.

(a) $f(z) = x^2 + y^2 + i\,2xy$

$$\frac{\partial}{\partial x}(x^2+y^2) \stackrel{?}{=} \frac{\partial}{\partial y}\,2xy$$

$$2x = 2x \quad \checkmark$$

$$\frac{\partial}{\partial y}(x^2+y^2) \stackrel{?}{=} -\frac{\partial}{\partial x}\,2xy$$

$$2y \stackrel{?}{=} -2y \quad \text{Nope Not diff'b'l.}$$

(b) $f(z) = \cos x \cosh y + i(-\sin x \sinh y)$

$$\frac{\partial}{\partial x}\cos x \cosh y \stackrel{?}{=} \frac{\partial}{\partial y}(-\sin x \sinh y)$$

$$-\sin x \cosh y = -\sin x \cosh y \quad \checkmark$$

$$\frac{\partial}{\partial y}\cos x \cosh y \stackrel{?}{=} -\frac{\partial}{\partial x}(-\sin x \sinh y)$$

$$\cos x \sinh y = +\cos x \sinh y \quad \checkmark$$

yes this satisfies C-R.

(4)

4 Parameterize and eval

a) $\int_\Gamma z\,dz \qquad \Gamma = \{(1+i)t : 0 \le t \le 1\}$

$= \int_0^1 ((1+i)t)(1+i)\,dt$

$= (1+i)^2 \int_0^1 t\,dt$

$= \boxed{\frac{1}{2}(1+i)^2}$

b) $\int_\Gamma |z|\,dz$

$= \int_0^1 |(1+i)t|\,(1+i)\,dt$

$= \sqrt{2}(1+i)\int_0^1 t\,dt$

$= \boxed{\sqrt{2}(1+i)\frac{1}{2}}$

(5)

c) $\int_\Gamma z^2\,dz \qquad \Gamma = \{ e^{it} : 0 \le t \le \frac{\pi}{2} \}$

$$= \int_0^{\pi/2} (e^{it})^2\, i e^{it}\, dt$$

$$= i\int_0^{\pi/2} e^{3it}\, dt = \frac{e^{3it}}{3i}\Big|_0^{\pi/2}$$

$$= \frac{e^{3i\frac{\pi}{2}} - 1}{3} = \frac{(e^{i\pi/2})^3 - 1}{3}$$

d) $\int_\Gamma Re(z)\,dz \qquad Re(e^{it}) = \cos t$

$$= \int_0^{\pi/2} \cos t\; i e^{it}\, dt$$

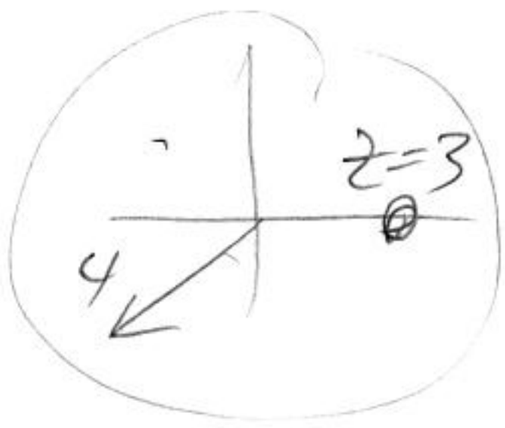
$$= i\int_0^{\pi/2} (\cos^2 t + i \cos t \sin t)\, dt$$

$$= i\left[\pi/4 + \sin(\pi)/4 \right] + \left[\frac{\sin^2(\pi/2)}{2} - 0 \right.$$

$$\boxed{-1/2 + i\pi/4}$$

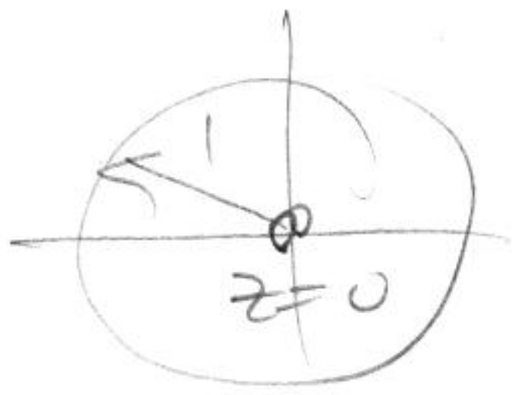
(6)

5 a) $\frac{1}{2\pi i} \oint_{|z|=4} \frac{z^2+5}{(z-3)^2} dz = \frac{d}{dz}(z^2+5)\Big|_{z=3}$



$\oint_{|z|=4} \frac{z^2+5}{(z-3)^2} dz = 2\pi i \cdot 6 = \boxed{12\pi i}$

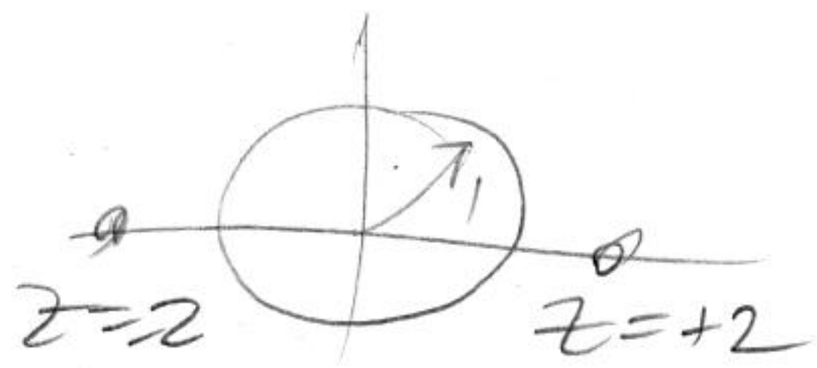
b) $\frac{1}{2\pi i} \oint_{|z|=1} \frac{\sin(z)}{z^2} dz = \frac{d}{dz} \sin(z)\Big|_{z=0}$



$\oint_{|z|=1} \frac{\sin(z)}{z^2} dz = \boxed{2\pi i}$

c) $\oint_{|z|=1} \frac{1}{(z-2)(z+2)} dz$

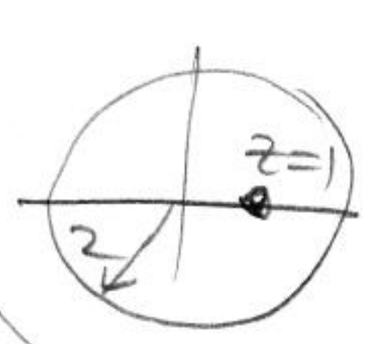
analytic.



$= \boxed{0}$

(7)

d) $\frac{2!}{2\pi i}\oint_{|z|=2} \frac{e^{2z}}{(z-1)^3}\,dz = \left.\frac{d^2}{dz^2}e^{2z}\right|_{z=1}$



$\oint_{|z|=2} \frac{e^{2z}}{(z-1)^3}\,dz = \pi i \cdot 4e^{2z}\Big|_{z=1} = \pi i \cdot 4e^2$

$\oint_{|z|=2} \frac{e^{2z}}{(z-1)^3}\,dz = 4\pi i e^2$