Math 3364 Exam 1. Sanders Fall 2026

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Find all solutions to the following.

(a) $z^4 = 1$ (b) $z^2 = 4i$

You may leave your answer in polar form with $\theta \in (-\pi, \pi]$ or if you want $\theta \in [0, 2\pi)$.

2. Determine the real and imaginary parts, $u(x, y)$ and $v(x, y)$, of the given complex functions $f(z)$.

(a) $f(z) = (z-1)^2$. (b) $f(z) = \dfrac{z}{z+1}$.

3. Is it possible for $f(z) = u(x, y) + i\, v(x, y)$ to be a differentiable (w.r.t. z) complex function for the given u and v?

(a) $u(x, y) = x^2 + y^2, \quad v(x, y) = 2xy.$

(b) $u(x, y) = \cos x \cosh y, \quad v(x, y) = -\sin x \sinh y.$

Recall, Cauchy-Riemann says: $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$.

4. Compute the line integral $\int_\Gamma f(z)\, dz$ explicitly by using the given parameterization when $f(z)$ and Γ are given as follows.

(a) $f(z) = z, \qquad \Gamma = \{\gamma(t) = (1+i)\, t : 0 \le t \le 1\}.$

(b) $f(z) = |z|, \qquad \Gamma = \{\gamma(t) = (1+i)\, t : 0 \le t \le 1\}.$

(c) $f(z) = z^2, \qquad \Gamma = \{\gamma(t) = e^{it} : 0 \le t \le \pi/2\}.$

(d) $f(z) = \mathrm{Re}(z), \ \Gamma = \{\gamma(t) = e^{it} : 0 \le t \le \pi/2\}.$

On (d), you may freely use $\int \cos^2(t)\, dt = t/2 + \sin(2t)/4$ and $\int \cos(t)\sin(t)\, dt = \sin^2(t)/2$.

5. Let C_ρ denote a circle of radius ρ centered at the origin, $z = 0$. Use Cauchy's integral formula to evaluate the following.

(a) $\displaystyle\oint_{C_4} \frac{z^2+5}{(z-3)^2}\, dz.$ (c) $\displaystyle\oint_{C_1} \frac{1}{(z-2)(z+2)}\, dz.$

(b) $\displaystyle\oint_{C_1} \frac{\sin(z)}{z^2}\, dz.$ (d) $\displaystyle\oint_{C_2} \frac{e^{2z}}{(z-1)^3}\, dz.$