1. Each given function \( f(z) \) has an isolated singularity at \( z = 0 \). Determine its full Laurent expansion about \( z = 0 \), classify the singularity type and state the residue value.
   
   (a) \( f(z) = \frac{\sin(z)}{z^2} \)
   
   (b) \( f(z) = \frac{e^{z^2} - 1}{z^5} \)  
   
   You may freely use \( \sin(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!} \) and \( e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \).

2. Each given \( f(z) \) has an isolated singularity at \( z = 0 \). Determine each term in the singular part of its Laurent expansion about \( z = 0 \), classify the singularity type and state the residue value.
   
   (a) \( f(z) = \cos(z) \sin(z) \)
   
   (b) \( f(z) = \frac{1}{e^z - 1 - z} \)

   You may use \( \cos(z) = 1 - z^2/2 + \cdots \).

3. Determine the location of all poles and also determine the pole’s order for the following.
   
   (a) \( f(z) = \frac{1}{z^2 + 1} \)
   
   (b) \( f(z) = \frac{z^2}{\sin(z)} \)

4. In this problem, you are asked to evaluate the value of the integral \( I \equiv \int_0^{2\pi} \frac{1}{2 + \cos \theta} \, d\theta \).
   
   (a) Write \( I \) as a complex integral \( \oint_{\Gamma} f(z) \, dz \) where \( \Gamma \) is parameterized by \( z(\theta) = e^{i\theta} \).
   
   (b) Determine the locations of all singularities of this \( f(z) \).
   
   (c) Determine \( f \)’s residue at the one singularity inside \( \Gamma \). BTW \(-1 < -2 + \sqrt{3} < 0\).
   
   (d) Use these results to compute the value of given real integral \( I \).

5. Consider the complex path integral \( \oint_{\Gamma_R} \frac{e^{iz}}{z^2 + 1} \, dz \) where \( \Gamma_R \) is the sideways "D" considered on your homework. (Ask me and I’ll specify \( \Gamma_R \) further on the blackboard.)
   
   (a) Determine the residue of the integrand at the singularity inside \( \Gamma_R \) for \( R > 1 \).
   
   (b) Use this result to determine the value of the real integral \( \int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 1} \, dx \).

Jordan’s lemma is not needed here to do part (b).