In §2.4 of your text it was shown that the acceleration of a particle can be written in polar coordinates as

\[ \ddot{a} = \left[ \frac{d^2r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 \right] \ddot{u}_r + \left[ r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right] \ddot{u}_\theta. \]

Kepler’s first law of planetary motion (i.e. a planet moves along an elliptical path with the sun at one of the foci) can be written in polar coordinates as

\[ r = \frac{a}{1 + e \cos \theta} \]

where \( a \) is the semifocal width and \( e < 1 \) is the eccentricity of the ellipse. Kepler’s second law (equal area swept out in equal time) can be written as

\[ r^2 \frac{d\theta}{dt} = \text{const.} \]

(a). Use the second law to deduce that the force between the sun and an orbiting planet is purely radial. That is, conclude that the \( \theta \)-component of the planet’s acceleration is zero. (Hint: Show that \( r\theta_{tt} + 2r_t\theta_t = (r^2\theta_t)_t \).

(b) Use the first and second laws together to conclude that the force is proportional to the inverse square of the distance between the sun and its orbiting planet. That is, show \( r_{tt} - r(\theta_t)^2 \sim 1/r^2 \). (Hint: Differentiate the expression \( r = a/(1 + e \cos \theta) \) twice with respect to \( t \) in order to conclude that \( r_{tt} = (r^2\theta_t)^2(1/r^3 - 1/ar^2) \). Use this to evaluate \( r_{tt} - r(\theta_t)^2 \).)