

Section 2.1: Solving Linear Programming Problems

Definitions:

An **objective function** is subject to a system of constraints **to be optimized** (**maximized or minimized**)

Constraints are a system of equalities or **inequalities** to which an **objective function is subject to.**

A **linear programming problem** consists of an objective function subject to a system of constraints

Example of what they look like:

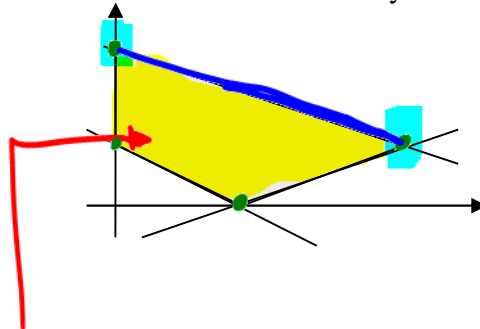
An **objective function** is $\max P(x,y) = 3x + 2y$ or $\min C(x,y) = 4x + 8y$

Constraints are: $5x + 3y \leq 120$
 $2x + 6y \leq 60$

A **linear programming problem** consists of a both the objective function subject to restraints.

$$\begin{aligned} \text{Max } P(x, y) &= 3x + 2y \\ \text{St: } x + y &\leq 4 \\ 2x + 5y &\leq 80 \\ x, y &\geq 0 \end{aligned}$$

Consider the following figure which is associated with a system of linear inequalities:



Definitions:

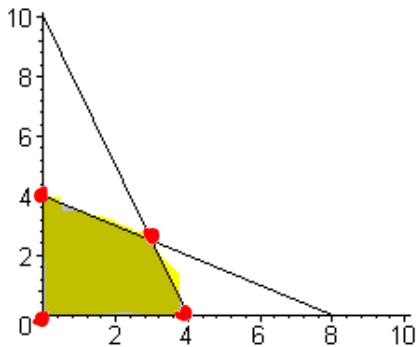
The region is called a **feasible set**. Each point in the region is a candidate for the **solution of the problem** and is called a feasible **solution**.

The point(s) in region that optimizes (maximizes or minimizes) the objective function is called the **optimal solution**.

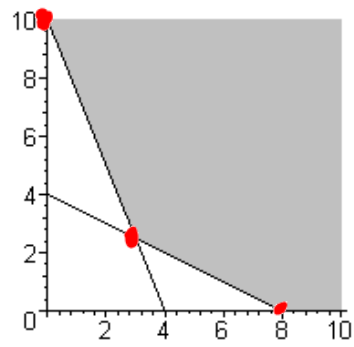
Fundamental Theorem of Linear Programming

- Given that an **optimal solution** to a linear programming problem exists, **it must occur at a vertex of the feasible set.**
- If the **optimal solution** occurs at two adjacent vertices of the feasible set, then the **linear programming problem has infinitely many solutions**. Any point on the line segment joining the two vertices is also a solution.

This theorem is referring to a solution set like the one that follows:



Maximum



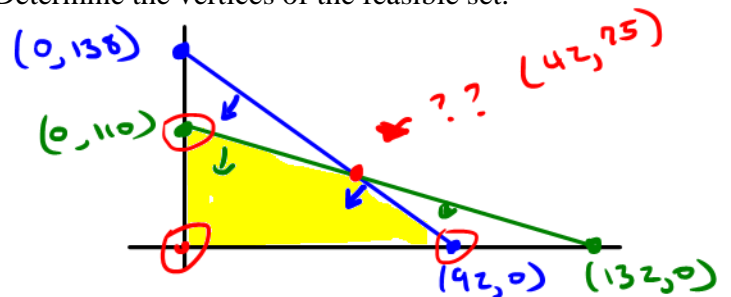
Minimum

The Method of Corners

1. Graph the feasible set.
2. Find the coordinates of all corner points (vertices) of the feasible set.
3. Evaluate the objective function at each corner points.
4. Find the vertex that renders the objective function a maximum (minimum). If there is only one such vertex, then this vertex constitutes a unique solution to the problem. If the objective function is maximized (minimized) at two adjacent corner points of S, there are infinitely many optimal solutions given by the points on the line segment determined by these two vertices.

Example 1: Given the following Linear Program, Determine the vertices of the feasible set.

Max profit $P(x,y) = 12x + 10y$
 $15x + 10y \leq 1380$ (1)
 Subject to: $10x + 12y \leq 1320$ (2)
 $x, y \geq 0$



Line 1
x-int
 $y = 0$
 $15x + 0 = 1380$
 $15x = 1380$
 $x = 92$

$15x + 10y \leq 1380$
 $10y \leq -15x + 1380$
 $y \leq -\frac{15}{10}x + \frac{1380}{10}$
 $y \leq -\frac{3}{2}x + 138$ *
 ↗ shade below

y-int
 $x = 0$
 $0 + 10y = 1380$
 $10y = 1380$
 $y = 138$

$10x + 12y \leq 1320$
 $12y \leq -10x + 1320$
 $y \leq -\frac{10}{12}x + \frac{1320}{12}$
 $y \leq -\frac{5}{6}x + 110$ *
 ↗ shade below

Line 2
x-int
 $y = 0$
 $10x + 0 = 1320$
 $10x = 1320$
 $x = 132$

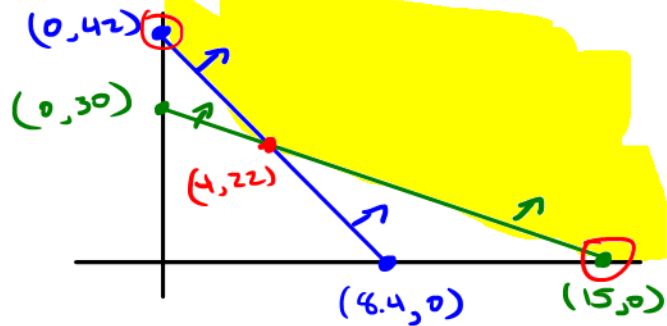
y-int
 $x = 0$
 $0 + 12y = 1320$
 $12y = 1320$
 $y = 110$

$-\frac{3}{2}x + 138 = -\frac{5}{6}x + 110$
 $-\frac{4}{6}x = -28$
 $x = 42$
 $y = 75$

$(0,110), (42,75)$
 $(0,0), (92,0)$

Example 2: Given the following Linear Program, Determine the vertices of the feasible set

Min $D_3 = 3x + y$
 $10x + 2y \geq 84$ (1)
 Subject to: $8x + 4y \geq 120$ (2)
 $x, y \geq 0$



Line 1
 $\frac{x\text{-int}}{y=0}$ $\frac{y\text{-int}}{x=0}$
 $10x = 84$ $2y = 84$
 $x = 8.4$ $y = 42$

Line 2
 $\frac{x\text{-int}}{y=0}$ $\frac{y\text{-int}}{x=0}$
 $8x = 120$ $4y = 120$
 $x = 15$ $y = 30$

$10x + 2y \geq 84$

$8x + 4y \geq 120$

$2y \geq -10x + 84$

$4y \geq -8x + 120$

$y \geq -\frac{10}{2}x + \frac{84}{2}$

$y \geq -\frac{8}{4}x + \frac{120}{4}$

$y \geq -5x + 42$

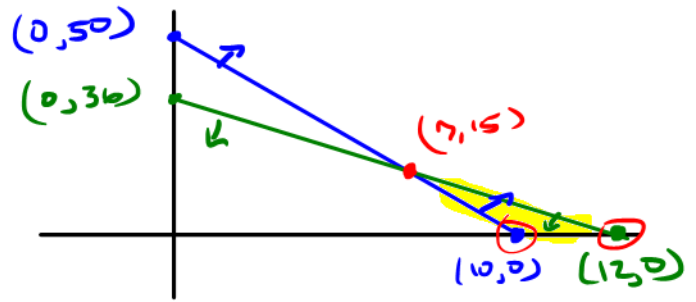
$y \geq -2x + 30$

$-2x + 30 = -5x + 42$
 $3x = 12$
 $x = 4$
 $y = 22$

- $(0, 42)$
- $(4, 22)$
- $(15, 0)$

Example 3: Given the following Linear Program, solve for the optimal solution.

Min $C(x, y) = 1200x + 100y$
 $40x + 8y \geq 400$ (1)
 Subject to: $3x + y \leq 36$ (2)
 $x, y \geq 0$



Line 1

x-int y-int
 $y = 0$ $x = 0$
 $40x = 400$ $8y = 400$
 $x = 10$ $y = 50$

$40x + 8y \geq 400$

$8y \geq -40x + 400$

$y \geq -5x + 50$

$-5x + 50 = -3x + 36$

$-2x = -14$

$x = 7$

$y = 15$

Line 2

x-int y-int
 $y = 0$ $x = 0$
 $3x = 36$ $y = 36$
 $x = 12$

$3x + y \leq 36$

$y \leq -3x + 36$

Pts	Min $C(x, y) = 1200x + 100y$
$(7, 15)$	$1200(7) + 100(15) = 9,900$
$(10, 0)$	$1200(10) + 100(0) = 12,000$
$(12, 0)$	$1200(12) + 100(0) = 14,400$

Optimal Value is 9,900
 occurred $(7, 15)$

Example 4: Maximize the following Linear Programming Problem.

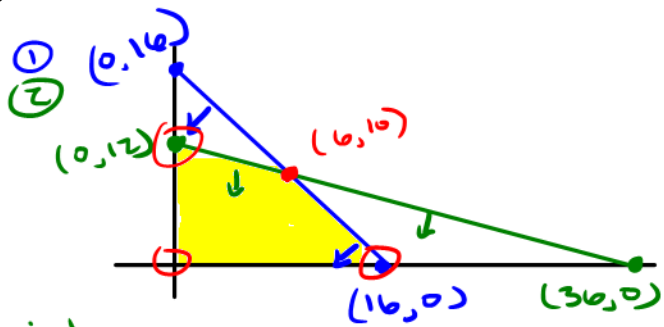
Maximize $C = 6x + 20y$

s.t. $x + y \leq 16$

$x + 3y \leq 36$

$x \geq 0$

$y \geq 0$



Line 1

x -int	y -int
$y = 0$	$x = 0$
$x = 16$	$y = 16$

Line 2

x -int	y -int
$y = 0$	$x = 0$
$x = 36$	$3y = 36$
	$y = 12$

$x + y \leq 16$
 $y \leq -x + 16$

$x + 3y \leq 36$
 $3y \leq -x + 36$
 $y \leq -\frac{1}{3}x + 12$

$-x + 16 = -\frac{1}{3}x + 12$
 $-\frac{2}{3}x = -4$
 $x = 6$
 $y = 10$

Pts	Max $C = 6x + 20y$
$(0, 12)$	$6(0) + 20(12) = 240$
$(6, 10)$	$6(6) + 20(10) = 236$
$(16, 0)$	$6(16) + 20(0) = 96$
$(0, 0)$	$6(0) + 20(0) = 0$

Optimal solution occurs at $(0, 12)$ w/ value = 240

Example 5: Maximize the following Linear Programming Problem.

Maximize $C = 2x + 6y$

s. t. $x + 3y \leq 15$ (1)
 $4x + y \leq 16$ (2)
 $x \geq 0$
 $y \geq 0$

Line 1

x-int: $(15, 0)$

y-int: $(0, 5)$

$x + 3y \leq 15$

$3y \leq -x + 15$

$y \leq -\frac{1}{3}x + 5$

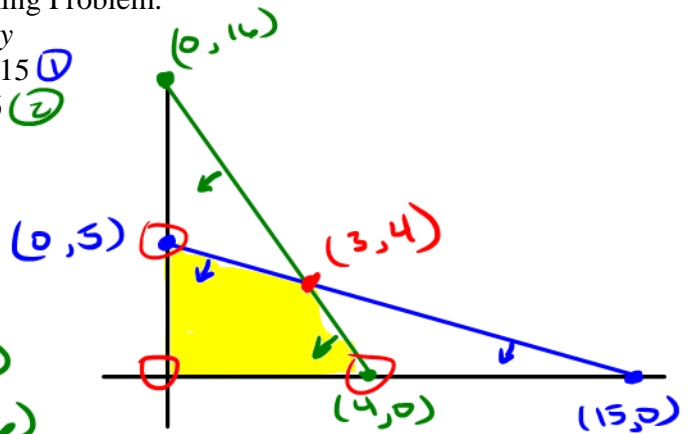
Line 2

x-int: $(4, 0)$

y-int: $(0, 16)$

$4x + y \leq 16$

$y \leq -4x + 16$



$-\frac{1}{3}x + 5 = -4x + 16$

$\frac{11}{3}x = 11$

$x = 3$

$y = 4$

pts	Max $C = 2x + 6y$
$(0, 5)$	$2(0) + 6(5) = 30$
$(3, 4)$	$2(3) + 6(4) = 30$
$(4, 0)$	$2(4) + 6(0) = 8$
$(0, 0)$	$2(0) + 6(0) = 0$

Infinite solutions