Foundations of Thinking

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One of the most critical issues in the study of mathematics is the foundation upon which we build our thinking. The mathematicians of old demonstrated for us the importance of building their conclusions upon a few accepted definitions, axioms or assumptions. Euclid built his Elements on five postulates and five axioms. Even in modern textbooks for high school geometry, one will often find in the first section of the first chapter the undefined terms: point, line and plane.

It is essential that we understand that our starting points are assumptions. They are statements that we cannot prove. Some have referred to them as presuppositions. Similarly, we can think about undefined terms. Just as in a dictionary one has to assume the meaning of the words used to define the first word; similarly, in mathematics, in order to prove our first theorem, we must have common, agreed upon definitions or assumptions.

It cannot be stated enough that these starting points are not provable. They are assumed or agreed upon. One could even say they are taken by faith. At the minimum, one must realize that any system built upon these assumptions is only as strong as the assumptions themselves. Should one assumption be shown to be false, then the whole system would need to be reworked from the beginning.

Consider these statements which demonstrate Aristotle’s thinking on this issue: “Aristotle intends us to understand that prior to the demonstrations in a scientific treatise, the treatise
should state starting propositions.” And again, “Aristotle has … separate concerns. One evolves from his argument that there must be first, unprovable principles for any science, in order to avoid both circularity and infinite regresses.”

It is worth noting that one can start with two or more different sets of assumptions and build multiple systems upon those assumptions. Starting with different assumptions will often lead, predictably, to different conclusions. Again, the important point of emphasis for this essay is that we recognize the existence of these assumptions and that our systems are only as reliable as their presuppositions.

One mathematical example of this scenario would be what is often called Euclidean and Non-Euclidean Geometry. In the *Elements*, Euclid’s fifth postulate essentially stated that parallel lines will not intersect. Due to the inability to explore these lines into infinity, Euclid took that statement as a postulate and built the basic system of geometry as we know it today. However, more recently, C.F. Gauss, working on the assumptions that these parallel lines might meet (consider how the longitudinal lines on a globe are parallel at the equator but intersect at the poles) developed what has been referred to as Non-Euclidean Geometry.

In the remainder of this essay, I would like to explore how we, in recent history and in popular science, seem to have forgotten that everyone starts with assumptions – everyone starts with presuppositions that must be accepted without proof. It seems to me that this is particularly prominent as it relates to our understanding of origins (e.g. the theory of evolution vs. creationism).

In recent years, in what I will call popular science, the theory of evolution has begun to be presented as an established fact, rather than as a theory that seeks to explain the origins of our

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existence. As our forefathers understood, the system or the theory is only as reliable as the foundation upon which it stands. Though it is an oversimplification, I would suggest that the theory of evolution (and the Big Bang theory) stands upon (among others) the assumption that matter is eternal. Without the eternality of matter, there is no “stuff” from which other “stuff” can evolve. If, however, matter is not eternal, then one needs to revisit the theory from its origins.

Another system, or theory, is based on the assumption that God is eternal and has created the world and everything in it.\(^2\) This presupposition, like the eternality of matter, is unprovable (despite many attempts to do so). Yet, the system of thinking that is built upon this assumption looks very different from the system built on the eternality of matter. One example would be in the realm of morality. If matter is eternal and, as is often times assumed, there is no God, then man is the measure of all things and he alone determines what is right and wrong. Yet, on the other hand, if an eternal God created the world, then he alone, as the creator has the right to determine what is right and wrong. The notable point for this essay is not whether one is a creationist or an evolutionist (or any other “origin-ist”), but instead, that everyone, often times unaware, starts with their own assumptions and from those assumptions builds a system of thinking. We are not being honest with ourselves if we think our system is such that its foundation is unable to be questioned.

Another variable that often enters the mix is influence of our own desires. Each one of us wants to believe that certain propositions are true. The Greeks wanted to believe that all numbers

\(^2\) It should be noted that this is the assumption that the author believes to be true. However, the point of the essay is not to debate about our assumptions, but rather to recognize that everyone starts with an assumption and therefore our systems are only as strong as the presuppositions upon which they are built.
were rational. To that end, they “were greatly shocked when they discovered irrational lengths, such as $\sqrt{2}$. At first they tried to keep this discovery secret.”\textsuperscript{3} It seems that even the one who discovered this truth met his demise because of his “inconvenient” discovery. Another example was Galileo and his proposition that the sun was the center of the galaxy. Sadly, many today, in the scientific community, set forth the theory of evolution as if there are no other options and this is to be regretted.

Edwin Abbott, in his mathematical novel \textit{Flatland}\textsuperscript{4} nicely illustrates how what we want to believe often overshadows our ability to think clearly. The fictitious characters in \textit{Flatland}, even when confronted with certain realities, sought to reject those realities for their own desires. Certainly, we are prone to this same temptation in our own day.

At the end of the day, we can thank the mathematicians of antiquity for their insistence that we start with, and even state, our underlying, unprovable assumptions. To act as if we do not have any, or as if our systems are above such faith-like assumptions, is simply to be dishonest.