Leonardo of Pisa, nicknamed Fibonacci, was an Italian mathematician who wrote several books that contributed to the study of mathematics. These books introduced Europe to Indian and Arabian mathematics. Fibonacci was born in 1170 in Pisa to his father, Guilielmo, who held a diplomatic post representing merchants of the Republic of Pisa. Because of his father’s job, Fibonacci was able to travel to many countries where he was introduced to different mathematical systems. He was sent to Bugia to study calculation and later went to Egypt, Syria, Greece, Sicily, and Provence, where he studied different numerical systems. In his book Liber abaci, Fibonacci wrote, “…when I had been introduced to the art of the Indians’ nine symbols through remarkable teaching, knowledge of the art very soon pleased me above all else…” He returned to Pisa in 1200 and became renowned for the many books he wrote. Unfortunately, Liber abaci, Practica geometriae, and Liber quadratorum are the only texts that were not lost.3

Liber abaci, “The Book of Calculation,” is considered one of the most important mathematical books in the European Middle Ages, because it was the first Western book to introduce the Hindu-Arabic numeral system, which is the same number system that we use today. When Liber abaci was published in 1202, Europe was using Roman numerals, so Fibonacci translated some mathematical examples so Europeans could learn how to write numbers using the Hindu-Arabic system. This was very important to the development of modern

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mathematics because it allowed decimals using place value notation. Using these “new” numbers, Fibonacci wrote about the arithmetic he had learned, including addition and multiplication tables.\textsuperscript{4} Liber abaci also includes the first known description of trial division to test if a number was composite or prime. He also taught Europeans how to factor a composite number. Fibonacci wrote this book so Europeans could learn how to perform calculations without using an abacus. Thus, he included a section that had practical applications for commerce, such as converting currency, measurements, and calculating profits and interests. Fibonacci also derived approximations of irrational numbers and wrote proofs in Euclidean geometry. He also included a section that discussed how to solve other applied arithmetic and algebra problems, such as the Chinese remainder theorem, perfect numbers, and formulas for arithmetic series. In this section, he introduced a problem describing the growth of a population of rabbits. This was the origin of the Fibonacci sequence, for which Fibonacci is famous for.\textsuperscript{5}

The Fibonacci sequence is a series of Fibonacci numbers that starts with either 0,1… or 1, 1… and every number after that is the sum of the two preceding ones. The beginning of the Fibonacci sequence is:

\[1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…\]

Thus, Fibonacci numbers are noted as \(F_n\), where \(F_1 = 0\) or \(F_1 = 1\), and \(F_n = F_{n-1} + F_{n-2}\). In Liber abaci, Fibonacci starts with \(F_1 = 1\). The sequence appears in Indian mathematics, especially in the work of Virahanka, an Indian prosodist, when he compares variations of measurements. However, Fibonacci considers the growth of a rabbit population. The problem is:


\textsuperscript{5} Fibonacci, Liber Abaci.
“A man puts one pair of rabbits in a certain place entirely surrounded by a wall. How many pairs of rabbits can be produced from that pair in a year if the nature of these rabbits is such that every month each pair bears a new pair which from the second month on becomes productive?”

Although Fibonacci’s example was unrealistic, Fibonacci numbers appear in biological settings, such as branching in trees, the arrangement of leaves on a stem, the fruit sprouts of a pineapple, and the flowering of an artichoke. In fact, Fibonacci numbers appear so unexpectedly in biology that a journal, *Fibonacci Quarterly*, has been created to study this sequence.

*Liber abaci* became so well-known that in the 1220s, Holy Roman emperor Frederick II invited Fibonacci to study with other members of Frederick’s scientific entourage. This led Fibonacci to write *Liber quadratorum*, or “The Book of Square Numbers” in 1225. This book is considered Fibonacci’s masterpiece as it majorly contributed to number theory. Fibonacci used a combination of already proved theorems and theorems he created to write his own proofs that solved general solutions. In this book he found an inductive method for finding Pythagorean triples. First, he constructed square numbers into sums of odd numbers using the formula \( n^2 + (2n+1) = (n+1)^2 \). Then, as Fibonacci wrote,

“…when I wish to find two square numbers whose addition produces a square number, I take any odd square number as one of the two square numbers and I find the other square

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6 Fibonacci, *Liber Abaci*.
8 J J O’Connor, *Leonardo Pisano Fibonacci*. 
number by the addition of all the odd numbers from unity up to but excluding the odd square number.”

This method was a precursor to Euler’s works. Fibonacci also stated that $x^2 + y^2$ and $x^2 - y^2$ could not both be perfect squares, which greatly impacted the calculation of the area of rational right triangles. He also defined a congruum, which is a number of the form $ab(a + b)(a - b)$, if $(a + b)$ is even, or $4(ab(a + b)(a - b))$ if $(a + b)$ is odd. Also, Fibonacci proved that a square cannot be a congruum. This is only a portion of the theorems and proofs found in Liber quadratorum.

Although Fibonacci was initially famous for introducing the Hindu-Arabic numeral system in Liber abaci, his contributions to mathematics have been vastly overlooked. In 1240, he was recognized and granted a salary for advising the citizens on accounting matters. However, his work in number theory was virtually ignored during the Middle Ages. Modern mathematicians know his name because of the famous Fibonacci sequence, but still do not understand the substantial impact Fibonacci had on number theory.

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9 Fibonacci, Liber Abaci.
10 R. B. McClenon, Leonardo of Pisa and His Liber Quadratorum (1919).
Works Cited:


