TGTC 2012 – University of Houston

February 17–19

Abstracts

Friday evening

5:00–6:00 pm: Tom Mrowka (MIT)

Instantons and their impact on low dimensional topology (talk for graduate students)

In 1983 Donaldson shocked the mathematical community by using moduli spaces of instantons to show that certain topological four-manifolds admitted no smooth structure. The existence of these topological four-manifold was only proved a year earlier by Michael Freedman. Thus began a lively interaction between low dimensional topology and physics which has lead to a resolution of many old problems in low dimensional topology. This talk will survey some of the (many) highlights of this story.

8:00–9:00 pm: Tom Mrowka (MIT)

Instanton Floer homology for trivalent graphs

In the past few years Kronheimer and I have been reinvestigating various versions of instanton Floer homology for knots or links in three manifolds. These Floer groups are finite dimensional vector spaces associated to an oriented three manifold containing a link whose components are labeled by partial flag manifolds for $\mathbb{C}^n$. These groups are functorial in a natural way. A couple of years ago we used the $\mathbb{CP}^1 = Gr_1(\mathbb{C}^2)$ version to prove that Khovanov homology detects the unknot. More recently we used this same version to prove that Rasmussen's s-invariant could not detect exotic homotopy 4-spheres as was hoped by Gompf-Morrison-Walker-Freedman. We've now understood how to generalize this theory to knotted trivalent graphs and are beginning to understand how the $\mathbb{CP}^{n-1}$ version relates to Khovanov-Rozansky homology. This talk will discuss some of these developments.

Saturday

9:00–10:00 am: Jian Song (Rutgers University)

Geometric surgery by partial differential equations

We will discuss recent developments on how geometric partial differential equations can perform canonical geometric surgery. We propose the analytic minimal model program with Ricci flow to classify algebraic varieties via geometric surgeries in Gromov-Hausdorff topology equivalent to birational surgeries such as contractions and flips. We carry out the conjectural program for Kahler surfaces and projective varieties with mild singularities by combining techniques from PDE, pluripotential theory, Riemannian geometry and algebraic geometry. This approach is also applied to the study of degeneration of Calabi-Yau metrics. As an application, we prove a conjecture of Candelas and de la Ossa for conifold flops and transitions in relation to the string theory.

10:15–11:15 am: Timothy Perutz (University of Texas at Austin)

The arithmetic of homological mirror symmetry

This talk will be about an extremely simple symplectic manifold: the 2-torus, T. The story of homological mirror symmetry (HMS) for T, predicted by Kontsevich (1994), developed by Polishchuk–Zaslow (1998) and completed by Polishchuk (2000), describes a relation between curves inscribed on T and coherent sheaves on a 'mirror' family of elliptic curves. In down-to-earth form, it expresses the multiplication rules for theta-functions in terms of areas of triangles.

I will describe joint work with Yanki Lekili (ArXiv: 1102.3160 and work in progress) concerning a new *arithmetic* variant of HMS. We observe that the Fukaya category of the once-punctured 2-torus T_0 is defined over the integers, and infer that the mirror variety, if it exists, should be a curve also defined over the integers. We use deformation-theoretic techniques to prove that there really is a mirror variety, and identify it as the nodal Weierstrass curve $Y^2Z+XYZ = X^3$. More generally, we show that a 'relative' Fukaya category, for T with a marked point, is derived-equivalent over $\mathbb{Z}[[q]]$ to perfect complexes of sheaves on the Tate curve.

11:30 am –12:30 pm: Roland Glowinski (University of Houston)

On the nodal lines of the eigenfunctions of the Laplace-Beltrami operator for bounded surfaces in $\mathbb{R}^3$: a computational approach

ON THE NODAL LINES OF THE EIGENFUNCTIONS OF THE LAPLACE-BELTRAMI OPERATOR FOR BOUNDED SURFACES IN $\mathbf{R}^3$: A COMPUTATIONAL APPROACH

A. Bonito[a] & R. Glowinski[b]

Abstract: When the authors of this presentation started investigating, via a Scientific Computing approach, the properties of the ***nodal lines***[c] of the ***eigenfunctions*** of the ***Laplace-Beltrami operator***, for some bounded surfaces in $\mathbf{R}^3$, their main motivation was to look at the way these lines intersect (or don't), depending of the symmetries the surface verifies. Actually, one of the drivers of these computations was to extend investigations we did few years ago concerning properties of the nodal lines of the solutions of linear and nonlinear eigenvalue problems for bounded domains of $\mathbf{R}^2$, with and without symmetries. Actually, we quickly realized that visualization of the nodal lines of the eigenfunctions of the Laplace-Beltrami operator has important applications in ***Medical Imaging***, ***Brain Imaging*** in particular, which is clearly an important motivation to further investigate this exciting area of Computational Spectral Geometry. In this lecture we will briefly discuss the finite element based numerical methodology used to approximate the following Laplace-Beltrami eigenproblem (formulated here ***variationally***, which is very convenient for Galerkin-type approximations):

$$\begin{cases} \{u,\lambda\}\in H^1(\Sigma)\times\mathbf{R}, \\ \int_\Sigma \nabla_\Sigma u.\nabla_\Sigma v\, d\Sigma = \lambda\int_\Sigma uv\, d\Sigma, \forall v\in H^1(\Sigma), \\ u\neq 0, \end{cases} \tag{EIGP}$$

where ∇_Σ is the tangential at Σ gradient operator and $H^1(\Sigma)$ is the ***Hilbert-Sobolev space*** defined by

$$H^1(\Sigma) = \{v \mid v\in L^2(\Sigma),\ \int_\Sigma |\nabla_\Sigma v|^2\, d\Sigma < +\infty\}.$$

(EIGP) has been solved numerically for the sphere (for which analytical solutions exist, allowing comparisons between exact and approximate solutions), torus, and surfaces with few or no symmetries. The results of these numerical experiments will be presented during this lecture, raising questions of (pure) mathematical interest (the figure below corresponds to the first eigenfunction of the Laplace-Beltrami operator for which the nodal lines intersect, the surface Σ being defined by $1024\, xyz(1-x-y-z) = 1$).

[a] Texas A & M University, Department of Mathematics, College Station, Texas
[b] University of Houston, Department of Mathematics, Houston, Texas
[c] That is the lines on Σ which these functions vanish

It is worth mentioning that the geometrical properties of the nodal lines of the Laplace-Beltrami operator provide useful information on the location of the actuators if one aims to control a diffusion phenomenon taking place on $\sum$.

2:15–3:15 pm: Susan Tolman (University of Illinois at Urbana-Champaign)

Toric Integrable Geodesic Flows in Odd Dimensions

The geodesic flow on a compact, connected n-dimensional Riemannian manifold Q is **toric integrable** if there is an effective symplectic action of the torus $\mathbf{T}^n$ on $T^*Q \setminus Q$ that commutes with dilations and with the geodesic flow. There are only a few known examples of manifolds with toric integrable geodesic flows: compact tori, the two-sphere, the real projective plane, and lens spaces. A conjecture by Lerman states that these are the only possible examples. We prove this conjecture (up to homeomorphism) if $n \neq 3$ is odd or if $\pi_1(Q)$ is infinite. Additionally, if $n \neq 3$ is odd, or if $\pi_1(Q)$ is infinite, we show that the cosphere bundle of Q is equivariantly contactomorphic to the cosphere bundle of the torus $\mathbb{T}^n$.

3:30–4:30 pm: Igor Zelenko (Texas A&M University)

Geometry of filtered structures on manifolds: Tanaka's prolongation and beyond

My talk is devoted to local equivalence problem for vector distributions (subbundles of tangent bundles) on manifolds with respect to the action of the group of diffeomorphisms. Vector distributions appear naturally in Geometric Control Theory (as control systems linear with respect to control parameters) and Geometric Theory of Differential Equations (as natural distributions on submanifolds of jet spaces).

The general way to solve such equivalence problems is to assign to a geometric structure the (co)frame (or the structure of absolute parallelism) on some (fiber) bundle over the ambient manifold in a canonical way.

In my talk first I will review the classical approaches to this problem, making special emphasis to the algebraic version of Cartan's method of equivalence developed by N. Tanaka in 1970s. The central object in the Tanaka approach is the notion of a symbol of a distributions at a point, which is a graded nilpotent Lie algebra. The prolongation procedure (i.e. the procedure of getting a canonical frame) can be described in terms of natural algebraic operation in the category of graded Lie algebras.

Through this review of Tanaka theory I will motivate the recent approach of B. Doubrov and myself to this problem. Our approach is a combination of a cerain symplectification of the problem (taking its origin in Pontryagin theory in Optimal Control) and various novel Tanaka type prolongations.

This approach allowed us to make a unified construction of canonical frames for distribution of arbitrary rank independently of their Tanaka symbols, avoiding the problem of classification of graded nilpotent Lie algebras with given number of generators, which is important for the application of the Tanaka theory. Our approach significantly extends the set of distributions for which the canonical frame can be explicitly constructed.

4:45-5:45 pm: Andrew Putman (Rice University)

Representation stability, congruence subgroups, and mapping class groups

The homology groups of many natural sequences of groups $\{G_n\}_{n=1}^{\infty}$ (e.g. general linear groups, mapping class groups, etc.) stabilize as $n \to \infty$. Indeed, there is a well-known machine for proving such results that goes back to early work of Quillen. Church and Farb discovered that many sequences of groups whose homology groups do not stabilize in the classical sense actually stabilize in some sense as representations. They called this phenomena *representation stability*. We prove that the homology groups of congruence subgroups of $\mathrm{GL}_n(R)$ (for almost any reasonable ring R) and mapping class groups of manifolds with marked points satisfy a strong version of representation stability that we call *central stability*. The definition of central stability is very different from Church-Farb's definition of representation stability (it is defined via a universal property), but we prove that it implies Church-Farb's definition of representation stability. Our main tool is a new machine analogous to the classical homological stability machine for proving central stability.

Sunday

9:00–10:00 am: Simon Brendle (Stanford University)

Uniqueness theorems for constant mean curvature surfaces

A classical theorem due to Alexandrov asserts that the round spheres are the only embedded surfaces in Euclidean space which have constant mean curvature. I will discuss similar uniqueness theorems for constant mean curvature surfaces when the ambient space is a warped product manifold. In particular, our results apply to the Schwarzschild and Schwarzschild-deSitter manifolds, which are of interest in general relativity.

10:30–11:30 am: Gordon Heier (University of Houston)

Effective finiteness of deformation types of non-isotrivial families

In complex geometry and number theory, negative curvature implies many interesting and significant finiteness and degeneracy theorems. In complex geometry, an early result in this direction was, for example, the 1893 theorem of Hurwitz stating that the automorphism group of a compact Riemann surface of genus $g \geq 2$ has order no greater than $84(g-1)$. Analogous finiteness theorems for maps in higher dimensions were later proved by many authors. In number theory, the Mordell Conjecture (1922) and Shafarevich Conjecture (1962) have inspired a vast body of research in this spirit, including the seminal work by Faltings.

The lecture will begin with a general introduction and survey of these types of theorems. A basic formulation of the Shafarevich Conjecture, proven by Parshin and Arakelov, is as follows. Let B be a smooth complex projective curve and $S \subset B$ a finite subset. Let $q \geq 2$ be an integer. Then there are only a finite number of isomorphism classes of nonisotrivial minimal families $f : X \to B$ of curves of genus q with X smooth such that $f : X \backslash f^{-1}(S) \to B \backslash S$ is a smooth family. An effective version has since been proven by the speaker.

The main focus of the lecture will be on the higher dimensional case, where the most obvious generalization of the Shafarevich Conjecture no longer holds true. Instead, deformation types of families have to be considered. Our main theorem will be the following. Let B be a smooth complex projective curve of genus g, and $S \subset B$ be a finite subset of cardinality s. There exists an effective upper bound on the number of deformation types of admissible families of manifolds with negative first Chern class of dimension n with canonical volume v over B and prescribed degeneracy locus S. The effective bound only depends on the invariants g, s, n and v. The key new ingredient which allows for this kind of result is a careful study of effective birationality for families of manifolds with negative first Chern class. This is joint work with S. Takayama.