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Characteristic cohomology of the horizontal subbundle on flag manifolds and connections to Hodge theory

A (complex) flag manifold is a homogeneous manifold G/P with G a complex semisimple Lie group. (For example, the grassmannian $\mathrm{Gr}(k,n)$ of k-planes in n-space is homogeneous under the action of $G = \mathrm{SL}(n,\mathbb{C})$.) The horizontal subbundle H is the unique, minimal, homogeneous, bracket-generating subbundle of the tangent bundle $T(G/P)$.

The horizontal subbundle determines a quotient of the de Rham complex on X, and the characteristic cohomology (CC) is the cohomology of this quotient complex. One can think of the CC as the cohomology that induces ordinary cohomology on an integral X of H by virtue of X being a solution to a system of PDE.

The CC can be realized as the cohomology of a complex of differential operators. (This complex is related to the Rumin complex and its cousins.) Basic questions for such a complex are: When is the cohomology finite dimensional? When does it vanish? When does a local Poincaré lemma hold? I will discuss these questions and, if time allows, a connection with Hodge theory.