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Conformality and Invertibility in $\mathbb{R}^n$

We study the question of estimating the cardinality of a prescribed fiber of a locally invertible map. Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be a local diffeomorphism, $n \geq 3$, and $q \in F(\mathbb{R}^n)$. Using geometric, topological and analytic arguments, we show that if the pre-image of every 2-plane π containing q is a surface conformally diffeomorphic to $\mathbb{R}^2$ – relative to the notion of angle on $F^{-1}(\pi)$ inherited from the Euclidean inner product of $\mathbb{R}^n$, then the point q is assumed exactly once by F. The analogous result in two dimensions fails. In fact, every non-injective local diffeomorphism $\mathbb{R}^2 \to \mathbb{R}^2$ provides a counterexample. On the other hand, if the pre-image of every such π is only assumed to be conformal to a finitely punctured plane (the number of punctures depending on the plane), then q is assumed at most twice.