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Hermitian manifolds with flat connections

Given a compact Hermitian manifold, there are many metric connections that are unqiuely determined by the metric. The most famous ones include Chern connection, Bismut connection, and Levi-Civita (Riemannian) connection. When the metric is Kahler, all three agrees, while when the metric is not Kahler, they are all distinct. Different connections give different curvature tensors, and it would be natural to try to understand the "space forms" with respect to each connection. In particular, one would like to understand the flat cases as a starting case.

For the Chern connection, the result of Boothby in 1958, following the work of H-H Wong on complex parallelizable manifolds, classified all compact Hermitian manifolds with flat Chern connection. They are exactly the compact quotients of complex Lie groups equipped with left invariant metric.

In this talk, we would like to report some recent progress on the classification of compact Hermitian manifolds with flat Bismut connection or flat Levi-Civita connection. In the Bismut case, which is a joint work with Q. Wang and B. Yang, we were able to give a complete classification of such manifolds, as compact quotients of Samelson spaces, which are well understood. For the Levi-Civita connection, in a recent joint work with G. Khan and B. Yang, we were able to solve the 3 dimensional case, namely, we give a complete clasification of all complex structures on a flat 6-torus that are compatible with the flat metric. We will also discuss our recent joint work with B. Yang on Hermitian manifolds with flat Gauduchon connections.