Schedule for TGTC 2025

All talks are in 232 PGH.

Friday, April 25

5:00 – 5:45 PM

Eric Riedl (University of Notre Dame): ***Introduction to spaces of curves on varieties***
[talk for graduate students]

> Given a complex projective variety, a fundamental and important question about it is: what do the curves on the variety look like? The set of curves on the variety can be naturally viewed as an algebraic set itself, and this pretalk will give an introduction to the basics of these moduli spaces of curves, giving examples and highlighting their deformation theory.

6:00 – 7:15 PM

Light buffet in 646 PGH

7:30 – 8:30 PM

Eric Riedl (University of Notre Dame): ***Free curves in singular varieties***

> Rational curves play a critical role in understanding the birational geometry of varieties. Free curves are the easiest to work with, but on Fano varieties that are even mildly singular, it remains an open question whether these free rational curves exist. In this talk, we discuss free curves of higher genus. Using some ideas on stability of vector bundles, we show that any klt Fano variety has these higher-genus free curves. We then use the existence of these free curves to get some applications, including the existence of free rational curves in terminal Fano threefolds, the lengths of extremal rays of the cone of curves, and studying the fundamental group of the smooth locus of a terminal variety. This is joint work with Eric Jovinelly and Brian Lehmann.

Saturday, April 26

8:30 – 9:00 AM

Coffee, juice and doughnuts, in 646 PGH

9:00 – 10:00 AM

Brian Lehmann (Boston College): ***Homological stability for rational curves on degree 4 del Pezzo surfaces***

In 1979 Segal gave a surprising comparison between the spaces of algebraic and topological maps from $\mathbb{P}^1$ to $\mathbb{P}^n$: he showed that the homology of the algebraic maps "stabilizes" to the homology of the topological maps as the degree increases. This result has subsequently been extended to other many algebraic varieties. The expected behavior is captured by the Cohen-Jones-Segal conjecture.

Recently Das and Tosteson developed a new technique for proving the Cohen-Jones-Segal conjecture using birational geometry. I will report on ongoing work which establishes this conjecture for degree 4 del Pezzo surfaces. Time permitting, I will also discuss the applications of this technique to the number-theoretic analogue: Manin's Conjecture. This is joint work with Ronno Das, Sho Tanimoto, and Phil Tosteson.

10:30 – 11:30 AM

Lewis Bowen (UT Austin): ***Benjamini-Schramm convergence of high genus random translation surfaces***

Benjamini-Schramm convergence is a notion which captures the local geometry of a random point on a random space. It was originally introduced to study random rooted finite planar graphs (while sending the number of vertices to infinity), but it has since been generalized to a wide range of objects. A translation surface is a surface on which the local geometry is that of the Euclidean plane everywhere except for a discrete set of points called singularities. At each singularity, there is a multiple of 2π extra cone angle; that is, the local geometry is identical to the k-fold branched cover of the complex plane corresponding to the map $z \mapsto z^k$. The set of translation surfaces of genus g and area g admits a natural, Lebesgue-class, finite measure called Masur-Smillie-Veech (MSV) measure. In this talk, I will speak about joint work with Kasra Rafi and Hunter Vallejos where we prove Benjamini-Schramm convergence of MSV-distributed random translation surfaces as genus tends to infinity. We have also identified the limit, which is called a Poisson translation plane.

Lunch break

2:30 – 3:30 PM

Dusty Grundmeier (Ohio State University): ***Hilbert Functions and Rank Problems***

Let $r(z,\bar{z})$ be a real polynomial. The rank of r is given by the rank of the underlying matrix of coefficients. A natural problem is to study the rank of $r(z,\bar{z})\|z\|^2$. In this talk, we will discuss possible values of the ranks in several scenarios, including when $r(z,\bar{z})\|z\|^2 = \|h(z)\|^2$ for some holomorphic polynomial h. We will also describe an application to the degree estimates problem.

4:00 – 5:00 PM

Jiri Lebl (Oklahoma State University): ***CR functions at CR singularities: approximation, extension, and hulls***

Real submanifolds in complex spaces inherit a certain amount of complex structure. If such structure gives a vector bundle, we have a so-called CR submanifold. In that case, there is a fairly good understanding of the relationship of the solutions of the CR vector fields, the CR functions, and the restrictions of holomorphic functions to the submanifold. If the CR structure degenerates on the other hand, a lot less is understood. We will look at CR singular submanifolds and the problem of extension of CR functions as holomorphic functions. The first issue is defining what we mean by CR functions at singular points, and we will consider several possibilities, and several results on such holomorphic extensions.

6:00 PM

Restaurant Dinner at TBA:

Sunday, April 27

8:30 – 9:00 AM

Coffee, juice and doughnuts, in 646 PGH

9:00 – 10:00 AM

Yingying Wu (University of Houston): ***Comparison Theorems of Moduli of Trees and Dual Graphs over del Pezzo Surfaces***

We explore combinatorial correspondences between the dual graphs of lines over del Pezzo surfaces and the moduli space of trees. We establish explicit isomorphisms and embeddings of graphs, most notably, identifying the dual graph over quintic del Pezzo surfaces with PBHV4, which is the Petersen graph. Our main result, the Ladder Theorem, reveals a hierarchical embedding of dual graphs over del Pezzo surfaces into projectivized BHV spaces, uncovering a recursive structure across degrees. This framework suggests a broader discrete-geometric unification of moduli problems, inspired by ideas from the Langlands program.

10:30 – 11:30 AM

Zhizhang Xie (Texas A&M University): ***On Gromov's Dihedral Rigidity Conjecture of Scalar Curvature***

In this talk, I will present my joint work with Jinmin Wang and Guoliang Yu on a new index theorem for manifolds with singularities, such as manifolds with corners and, more generally, manifolds with polyhedral-type boundary. As an application, we obtained a positive solution to Gromov's dihedral rigidity conjecture. This conjecture concerns comparisons of scalar curvature, mean curvature and dihedral angles for compact manifolds with polyhedral-type boundary, and has very interesting implications in geometry and mathematical physics.

Further developments of this new index theorem have led us to a positive solution of Gromov's flat corner domination conjecture. As a consequence, we answered positively a long standing conjecture in discrete geometry - the Stoker conjecture.