Abstract.

1. True-False Problems

Throughout \((X,d)\) denotes a metric space.

(1) If \(d(p,q)=0\), then \(p\) and \(q\) are the same point.
(2) Every convergent sequence is bounded.
(3) Every bounded sequence is convergent.
(4) Every convergent sequence is Cauchy.
(5) Every Cauchy sequence is convergent.
(6) If \(C\) is closed and \(U\) is open, then \(C \cup U^c\) is closed.
(7) If \(S = \{1 + \frac{1}{n} : n \in \mathbb{N}\}\), then \(\inf S = 1\).

In the following problems \((\mathbb{R}^k,d)\) denotes Euclidean space.

(8) If \(p_n = (\frac{1}{n}, \frac{2n^2+1}{n^2+1})\) is a sequence in \(\mathbb{R}^2\), then this sequence converges and \(\lim_{n} p_n = (0, 2)\).
(9) If \(S = (0, 1) \cup (1, 2) \subseteq \mathbb{R}\) then \(\partial S = \{0, 1, 2\}\).
(10) If \(S = \{(x,0) : 0 < x < 1 \text{ or } 1 < x < 2\}\), then \(\partial S = \{(x,0) : 0 \leq x \leq 2\}\).

Throughout \((X,d)\) is a metric space.

2. Problem

Prove that the intersection of two compact sets is a compact set.

3. Problem

Let \(x, y, z \in X\). If \(d(x,y) = 11\), and \(d(x,z) = 7\), then what is the minimum possible value of \(d(y,z)\)? Prove your answer.

4. Problem

Let \(X = \mathbb{R}\) and define \(\rho(x,y) = |x - y| + |x^3 - y^3|\). Prove that \(\rho\) is a metric.
5. Problem

Let \((X, d)\) be a discrete metric space. Prove that \(X\) is complete.